A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 56 2011 Issue 4
DOI: 10.2478/v10172-011-0141-6

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PHOTOACOUSTIC DIAGNOSTICS OF INHOMOGENEOUS GYROTROPIC MATERIALS WITH INTERNAL STRESS USING BESSEL LIGHT BEAMS

FOTOAKUSTYCZNA DIAGNOSTYKA NIEJEDNORODNYCH ŻYROSKOPOWYCH MATERIAŁÓW Z WEWNĘTRZNYMI NAPRĘŻENIAMI Z ZASTOSOWANIEM LASEROWYCH WIĄZEK BESSELA

This paper considers the investigation of photoacoustic transformation in naturally-gyrotropic and magnetoactive crystals, with internal stress under sound excitation in different modes by Bessel light beams (BLB). In the range of high modulation frequencies ($\Omega > 1$ MHz), the dependence of the photoacoustic response amplitude on the radial coordinate ρ exhibits resonant phenomenon, which can be used to increase the resolution of photoacoustic spectroscopy for media with internal stresses.

The expressions for amplitudes of photoacoustic signals in strained crystalline samples were obtained under different boundary conditions, taking into account the dependence of the thermoelastic coupling coefficient on the initial strain in the sample. It was showed that a resonant increase in the amplitude signal is related to the dependence on the geometric parameters of the sample-piezoelectric transducer system, the values of Murnagan constants, the mode composition of the Bessel light beam, and its amplitude modulation frequency.

Keywords: photoacoustic spectroscopy, Bessel light beams, magnetoactive gyrotropic materials

W pracy przedstawiono wyniki badań transformacji fotoakustycznej poprzez dźwięk generowany laserowymi wiązkami Bessela o różnych modach w kryształach naturalnie żyrotropowych i magnetoaktywnych, z wewnętrznymi naprężeniami. Stwierdzono, że w zakresie modulacji o wysokiej częstotliwości ($\Omega > 1$ MHz), zależność amplitudy odpowiedzi fotoakustycznej od radialnej współrzędnej wykazuje efekt rezonansowy. Efekt ten może być wykorzystany do podwyższenia rozdzielczości spektroskopii fotoakustycznej w ośrodkach z wewnętrznymi naprężeniami.

Otrzymano wyrażenia na amplitudy sygnałów fotoakustycznych w próbkach krystalicznych z wewnętrznymi naprężeniami, przy różnych warunkach brzegowych. Brano przy tym pod uwagę zależność współczynnika sprzężenia termoplastycznego od wewnętrznych naprężeń w próbce. Wykazano, że rezonansowy wzrost amplitudy sygnału jest zależny od geometrycznych parametrów układu próbka – przetwornik piezoelektryczny, wartości stałych Murnagana, modów wiązki Bessela i częstotliwości modulacji.

1. Introduction

Method of laser photoacoustic spectroscopy has been broadly applicable lately for the investigation of interaction between electromagnetic radiation and different media. The use of laser sources in photoacoustics permitted to make a transition to qualitative higher level of measurement and to increase substantially sensitivity of the method has been demonstrated at investigation of media in different aggregative states in wide spectral range from ultraviolet to infrared, exhibition absorption both strong -10^5 cm^{-1} and very weak – 10^{-10} cm^{-1} [1–3].

And advanced method of photoacoustic spectroscopy along with conventional methods is applied at investigation of dissipative, thermal and nonlinear characteristics of magnetoactive and naturally gyrotropic media.

When studying the photoacoustic transformation in inhomogeneous and crystalline anisotropy, gyrotropy, and dichroism [4, 5]. Therefore, one of the main problems media, it is necessary to take into account the fact that the absorption of electromagnetic waves in them has

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a number of specific features which are caused, for example, by of laser photoacoustic spectroscopy is to find analytical solutions for inhomogeneous heat conduction equations, in which the right hand side contains the energy dissipation as a heat source power density. Having solved the electrodynamic boundary value problems using the covariant methods of direct tensor calculus (which were developed for the first time by F.I. Fedorov [4, 6]), one can easily determine the energy dissipation in media with an arbitrary combination of gyrotropy, dichroism, and anisotropy, for example, in crystals of middle syngonies [7], crystals with a cholesteric structure of anisotropy [8], naturally gyrotropic superlattices [9], etc.

Note that the approach to the problems of photoacoustic transformation, which is based on the solution of boundary value problems of electrodynamics by the covariant method and the subsequent calculation of the energy dissipation, is preferred over those traditionally used in photoacoustics [1, 6, 8], because it makes it possible to take into account (when necessary) boundary diffraction effects, multibeam interference, peculiarities of laser mode structure, etc.

The purpose of this study was to analyze naturally-gyrotropic and magnetoactive crystals with internal stress by laser photoacoustic spectroscopy under sound excitation by Bessel light beams (BLB) in different modes.

2. Magnetoactive samples

Let us consider the case of piezoelectric detection of a photoacoustic signal formed as a result of the absorption of the TE-mode of an amplitude modulated Bessel light beam by a crystal with internal stress (Fig. 1).

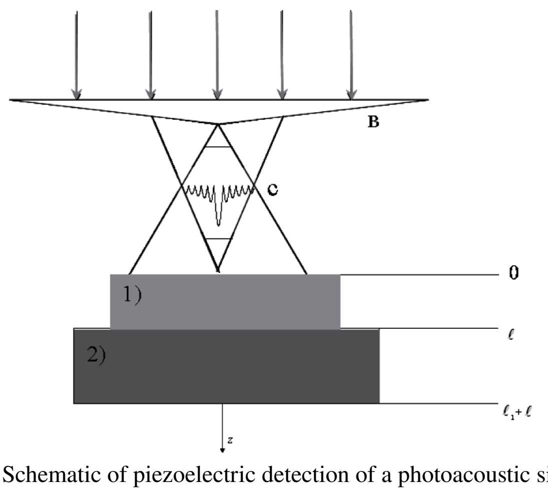

Fig. 1. Schematic of piezoelectric detection of a photoacoustic signal: (1) magnetoactive or gyrotropic sample, (2) piezoelectric sensor, (B) axicon, and (C) Bessel light beam

The properties of magnetoactive sample can be described with the help of material equations

$$\begin{gathered}\mathbf{E} = \left(\varepsilon^{-1} + i\mathbf{G}^X\right)\mathbf{D},^{\mathbf{E}} = G^{-1}\mathbf{H}, \\ \mathbf{B} = \mu\mathbf{H}, \mu = 1,^{G-1} = \left(\varepsilon^{-1} + iG^X\right)\end{gathered}, \qquad (1)$$

where $\mathbf{G}^X$ is antisimmetrical complex tensor of 2^{nd} rank, dual to vector of magnetic gyration $\mathbf{G}$, with the real part $Re\mathbf{G}^X = G'$ defining the specific rotation of polarization plane, while imaginary $Im\mathbf{G}^X = G''$ is responsible for value of magnetic circular dichroism, ε – dielectrical permittivity.

Considering vectors $\acute{\mathbf{L}}$ and $\hat{\mathbf{A}}$ being proportional $e^{i(k_z z + m\theta - \omega t)}$, from equation (1) and Maxwell equations in cylindrical coordinate system, we will come to the equations system for constituents vectors $\acute{\mathbf{L}}$ and $\hat{\mathbf{A}}$:

$$\begin{aligned}&1)\ \tfrac{im}{2\rho}E_z - ik_z E_\theta = ik_0 B_\rho,\\&2)\ ik_z E_\rho - \tfrac{\partial}{\partial\rho}E_z = ik_0 B_\theta,\\&3)\frac{1}{\rho}E_\theta + \frac{\partial}{\partial\rho}E_\varphi - \frac{im}{\rho}E_\rho = ik_0 B_z,\\&4)\ \tfrac{im}{2\rho}B_z - ik_z B_\varphi = -ik_0 G E_p,\\&5)\ ik_z B_\rho - \tfrac{\partial}{\partial\rho}B_z = -ik_0 G E_\varphi,\\&6)\ \tfrac{1}{\rho}B_\varphi + \tfrac{\partial}{\partial\rho}B_\varphi - \tfrac{im}{\rho}B_\rho = -ik_0 G E_z,\end{aligned} \qquad (2)$$

where ρ and θ – cylindrical coordinates,
$k_z = k_0\sqrt{\varepsilon}\cos\gamma$ – parameter BLB obliquity and equal to half of the corner at top of the cone of wave vectors, determining spectrum of spatial parts of BLB. Further it is simple to receive expression for energy dissipation of the TE-mode of BLB

$$Q^{TE} = \frac{c}{4\rho}\frac{q^2 m}{\rho\left|k_z^2 -_0^2 G\right|}\left\{\frac{k_{z2} Im\left(k_z^{2^*} -_0^2 G^*\right) Im\left(\frac{k_z^2 -_0^2 G}{k_z}\right)}{\left|\frac{k_z^2 -_0^2 G}{k_z}\right|}\right.$$
$$\left. iIm^2 n - 2Re\left(k_z^2 -_0^2 G\right) Re^2 n\right\} J'_m(qp)\, J_m(qp)\, e^{-2k_{z2}z}, \qquad (3)$$

where $J_m(qp)$ – is the m-th order Bessel function of the first kind,

$$J'_m(qp) = \frac{\partial J_m(qp)}{\partial\rho}. \qquad (4)$$

Distribution temperature field in magnetoactive media, absorbing amplitude-modulated BLB of TE-mode, can be described by inhomogeneous equation of thermal conductivity

$$\frac{\partial^2 T}{\partial z^2} - \frac{1}{\beta_s}\frac{\partial T}{\partial t} = -\frac{1}{2k_s}Q^{TE}(z), \qquad (5)$$

where T – temperature, β_s and k_s coefficients of thermal and temperature conductivity, related by expression $\beta_s = k_s/\rho_0 C$, ρ_0– sample solidity, C– specific heat.

Finding general and particular solution of equation (4), using stationary boundary conditions, expression for temperature field in absorbing magnetoactive sample

$$T(z) = \left(2\frac{Im(k_z)}{\sigma}e^{-\sigma z} + e^{-2Im(k_z)z}\right)\psi e^{i\Omega t}, \qquad (6)$$

$\psi = \frac{\tilde{Q}^{TE}}{2k_s(\sigma^2 - 4Im(k_z)^2)}$, $\sigma = (1+i)\sqrt{\frac{\Omega}{2\beta_s}}$– coefficient of thermal diffusivity, Ω – modulation frequency of BLB.

Distribution of temperature field (5) is necessary for calculation thermoelastic deformations in investigated sample and piezotranduser, and further for finding amplitude-phase characteristics of generated PA signal.

To determine deformation at non-linear parallax in terms of influence on body harmonically modulated on time laser radiation can be written following equation for elastic parallaxes [10, 11]

$$G_3^{(3)}\frac{\partial^2 \Delta u_3}{\partial z^2} = g^{(3)}\frac{\partial T(z,t)}{\partial z} + \rho_0 \Delta \ddot{u}_3, \qquad (7)$$

$G_3^{(3)} = t_{33}^{(0)} + b + 2(n+m)U_{33} + C_{33}, g^{(3)} = (1 + \vartheta U_{33})\gamma_0$, $b = 2\mu + (2m-n)U_{33}$, $C_{33} = K - \frac{2}{3}\mu + 2\ell_0 U_{33}$, ϑ – coefficient determining dependence of elastic connection from initial deformation, γ_0– coefficient of thermoelastic connection for a non deformated body, K– compressibility, m, n, ℓ_0– Murnagan constants, μ– coefficient Lame, U_{33}– multiplier of initial deformation vector, $t_{33}^{(0)}$– multiplier of tensor of initial stresses.

Solving equation (6) have expression for parallax parts of body, determined by deformation under influence modulated laser radiation at frequency Ω

$$\Delta u_3 = D_1 e^{-iQz} + D_2 e^{iQz} + Y \qquad (8)$$

where $Q = \sqrt{\frac{\rho_0\Omega^2}{G_3^{(3)}}}$, $Y = Y_1 e^{-\sigma z} + Y_2 e^{-2k_{z2}z}$,

$Y_2 = -\frac{2Im(k_z)g^{(3)}\psi}{G_3^{(3)}(4Im(k_z)^2 + Q^2)}$, $Y_1 = -\frac{g^{(3)}\psi 2Im(k_z)}{G_3^{(3)}(\sigma^2 + Q^2)}$.

Parallax borders of piezoelement can be found from differential expression for elastic parallaxes

$$\frac{\partial^2 u_3^{(p)}}{\partial z^2} - \frac{1}{v_1}\frac{\partial^2}{\partial t^2}u_3^{(p)} = 0, \qquad (9)$$

Solution of which is

$$u_3^{(p)}(z) = P_1 e^{-ik_1 z} + P_2 e^{ik_1 z}. \qquad (10)$$

Coefficients P_1 and P_2 are in following limited terms, for case of free borders

$$F(\ell) = F_1(\ell),\ \Delta u_3(\ell) = u_3^{(p)}(\ell),\ F(0) = 0,\ F_1(\ell + \ell_1) = 0, \qquad (11)$$

where $F(z) = c^T\frac{\partial \Delta u_3}{\partial z} - B\alpha_t \Delta T$ and $F_1(z) = c^D\frac{\partial u_3^{(p)}}{\partial z}$ – tensions; $c^T = \lambda + 2\mu$, λ – Lame coefficient; $c^D = c^E\left(1 + e^2 / \left(\varepsilon^s c^E\right)\right)$; e – piezomodule; c^E – inflexibility coefficient of piezoelectric; ε^s – pressed crystal permittivity; B – volumetric module of elasticity; α_t – coefficient of thermal volume extention.

Basing on methodics of works [11, 12, 13] we have expression for PA system response, taken from piezotranducer, at generation of thermoelastic signal in magnetoactive media by TE-mode of BLB

$$V = h\frac{\left(\frac{c^T Q\cos QL}{\sin QL}X_2(L) + X_1(L)\right)}{\left(c^D k_1\frac{\cos k_1 L_1}{\sin k_1 L_1} + \frac{c^T Q \cos QL}{\sin QL}\right)}, \qquad (12)$$

where

$$X(L) = c^T\frac{\partial Y}{\partial z}|_{z=L} - B\alpha_t T(L);$$

$$X_1(L) = X(L) + c^T iQY(0)e^{-iQL};$$

$$X_2(L) = Y(0)e^{-iQL} - Y(L);$$

L and L_1 – thickness of sample and piezoelement; $h = e/\varepsilon^s$; $k_1 = \frac{\Omega}{v_1}$; v_1 – sound speed in piezoelement.

As it's seen from expression (11) value of amplitude signal taken from piezoelement depends on dissipative and thermophysical properties of sample, parameter of magneto-circular dichroism and also on geometrical parameters of system "sample-piezoelement" and modulation frequency of radiation. Results of graphical analysis energy dissipation dependence on parameters ρ for different modes of BLB and also amplitude dependence of PA value on BLB modulation frequency and geometrical size of system "sample-piezoelement" are given in Fig. 1 and Fig. 2.

First let's investigate influence change of BLB radius on dependance energy dissipation in magnetoactive media from wave length of radiation using MathCad. For this let's choose media with the following parameters. $\mathbf{G} = 10^{-5} + i\cdot 10^{-7}$, $\varepsilon = 6.304 + i\cdot 2.56$ and BLB with $\gamma = 0.035$.

As it comes from graphs (Fig.2) on oscillation of energy dissipation is influenced by transversal spatial BLB structure, determined by Bessel functions of different ranks.

Changing stress value of external magnetoactive field, it is possible to influence on energy dissipation speed. As imaginary part of gyration parameter expressed through scalar product Verden constant and tensity. In this case maximums on graph of dependence Q^{TE} will shift, that will lead to displacement or appearance of resonance of PA signal amplitude in other spectral region. So there is a possibility not only of PA diagnostics of internal structure of magnetoactive media, but also to control amplitude-phase characteristics of PA signal.

Further analyze dependence of PA signal amplitude in elastic-stressed sample on radiation frequency modulation and geometrical parameters of system “sample-piezodetector”, which is described by equation (12).

It’s seen from graphs, that at optimal choice sample thickness and detector PA signal amplitude can increase by several times, which let increase resolution capability of laser PA spectroscopy. Also is necessary to point that on amplitude values of PA resonances is influenced by change of BLB radius and wave length of radiation.

Experimental measurement of resonance signals values taking in consideration expressions (12) let propose means the of determination of thermophysical, acoustical and dichroic parameters of absorbing elastic-stressed media by the method of laser PA spectroscopy.

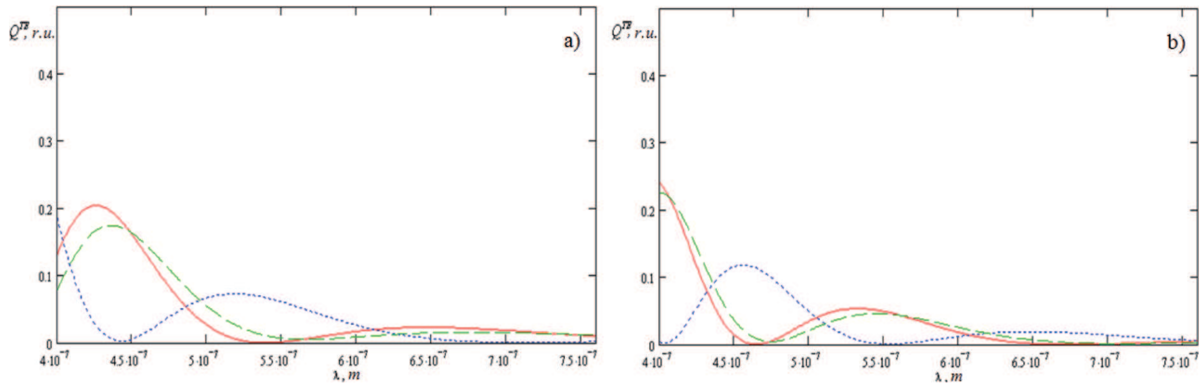

Fig. 2. Dependence of quantity distribution of absorbed heat in magnetoactive media Q^{TE} versus λ on radiation wave length and radial coordinate (r.u. – relative units) a) $\rho=8{\cdot}10^{-7}m$,b) $\rho=10^{-6}m$

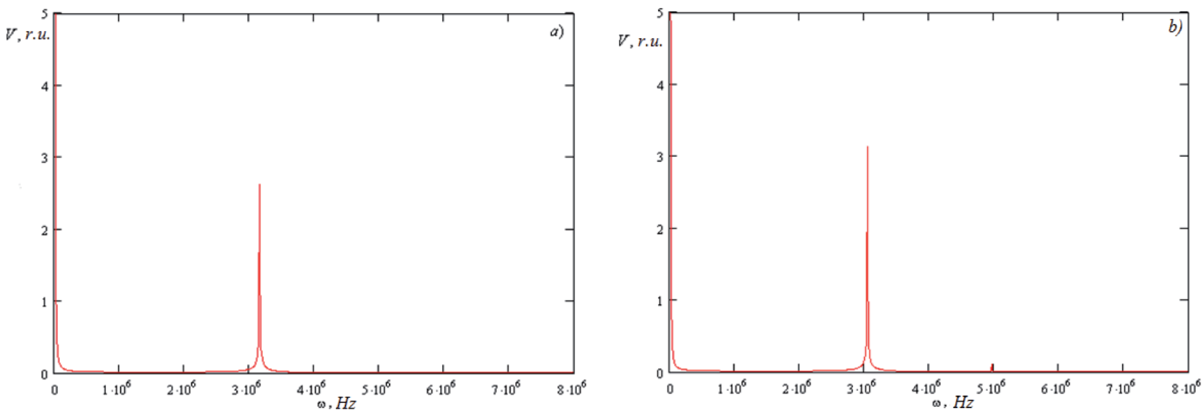

Fig. 3. Dependence of PA signal amplitude $V(\omega)$ in magnetoactive media on frequency modulation and longitudinal size of sample, $L_1 = 5 \cdot 10^{-4}$m a) $L = 5 \cdot 10^{-3}$m, b) $L = 7 \cdot 10^{-3}$m

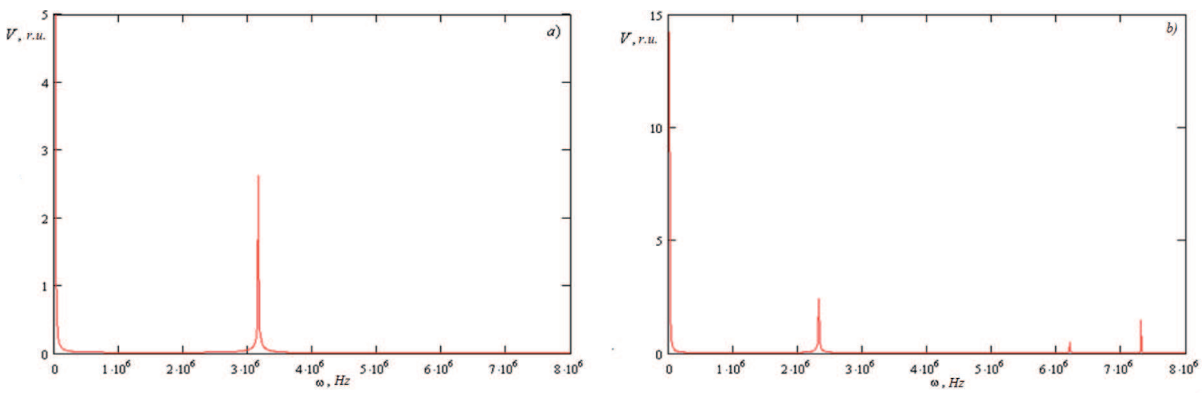

Fig. 4. Dependence of PA signal amplitude $V(\omega)$ in magnetoactive media on frequency modulation and longitudinal size of piezodetector, $L = 5 \cdot 10^{-3}$m a) $L_1 = 5 \cdot 10^{-4}$m, b) $L_1 = 7 \cdot 10^{-4}$m

3. Natural-gyrotropic sample

We proceed from the coupling equations [6, 14]

$$\vec{D} = \varepsilon\vec{E} + i\gamma\vec{H}, \quad \vec{B} = \mu\vec{H} - i\tilde{\gamma}\vec{E}, (\mu = 1) \tag{13}$$

where $\varepsilon = \varepsilon_1 + i\varepsilon_2$ is a dielectric tensor and $\gamma = \gamma_1 + i\gamma_2$ is the second-rank pseudo tensor, whose real part $\gamma_1 = Re\gamma$ determines the specific rotation of the plane of polarization and the imaginary part $\gamma_2 = Im\gamma$ responsible for natural circular dichroism.

Assuming that the vectors $\vec{D}$ and $\vec{B}$ are proportional to relation $\exp\left[i\left(k_z z + m\varphi - \omega t\right)\right]$ (1) and using Maxwell equations

$$rot\vec{E} = ik_o\vec{B},^r ot\vec{B} = -ik_o\vec{D}, \tag{14}$$

where $k_o = \omega/c$ – is the wavenumber, we can write a system of equations for the component of the vectors $\vec{E}$ and $\vec{B}$ in the chosen cylindrical coordinate system:

$$\frac{im}{\rho}E_z - ik_zE_\gamma = ik_0B_\rho,\ \frac{im}{\rho}B_z - ik_zB_\gamma = -ik_0\left(\varepsilon E_\rho + i\gamma B_\rho\right),$$

$$ik_zE_\rho - \frac{\partial}{\partial\rho}E_z = ik_0B\varphi,\ ik_zB_\rho - \frac{\partial}{\partial\rho}B_z = -ik_0\left(\varepsilon E_\varphi + i\gamma B_\varphi\right),$$

$$\frac{1}{\rho}E_\varphi + \frac{\partial}{\partial\rho}E_\varphi - \frac{im}{\rho}E_\rho = ik_0B_z, \tag{15}$$

$$\frac{1}{\rho}B_\varphi - \frac{\partial}{\partial\rho}B_\varphi - \frac{im}{\rho}B_\rho = -ik_0\left(\varepsilon E_z + i\gamma B_z\right),$$

where ρ, φ are cylindrical coordinates, $k_z = k_0\sqrt{\varepsilon}\cos\alpha = k'_{z_1} + ik''_{z_2}$ and α is the conicity parameter of the Bessel light beam, which is equal to a half of the angle at the apex of the cone formed by the wave vectors that determine the spatial frequency spectrum of the beam. The longitudinal components E_z and B_z, which satisfy the Helmholtz equation, can be written as follows:

$$E_z = c_zJ_m(q\rho)\exp\left[i\left(k_zz + m\varphi\right)\right],$$

$$B_z = b_zJ_m(q\rho)\exp\left[i\left(k_zz + m\varphi\right)\right], \tag{16}$$

where $J_m(q\rho)$ is the m-th order Bessel function of the first kind, $b_z = \mp iqn_\pm$, $c_z = q$, $n_\pm = \sqrt{\varepsilon}\pm\gamma$ is the complex refractive index of eigenwaves in the medium, and $q = k_0\sqrt{\varepsilon}\sin\alpha$. Assuming that and E_z are B_z specified, we express the other filed components in terms of these values and, based on the relations:

$$Q^{TE} = -\frac{\partial S_z^{TE}}{\partial z}, \tag{17}$$

$$S_z^{TE} = \frac{c}{8\pi}\left(E_\rho^{TE}B_\varphi^{TE*} - E_\varphi^{TE}B_\rho^{TE*}\right) + \kappa.c., \tag{18}$$

determine the energy dissipation of the TE-mode of Bessel light beam:

$$Q^{TE} = \frac{\omega|\varepsilon|\varepsilon_2}{2\pi}\left[\left(\frac{m}{q\rho}\right)^2 J_m^2(q\rho) + J'^2_m(q\rho) + \right.$$
$$\left. + \frac{2mk_0k_{z_1}\gamma_1}{q^3\rho}J_m(q\rho)J'_m(q\rho)\right]e^{-2k_{z_2}\cdot z} = \tilde{Q}^{TE}\exp\left(-2k_{z_2}\cdot z\right), \tag{19}$$

where $J'_m(q\rho) = \frac{\partial J_m(q\rho)}{\partial\rho}$.

The heat-source power density for the TH-mode is determined similarly; however, it is rather cumbersome and omitted for this reason.

The distribution of the temperature fields in the sample under study can be found by solving the inhomogeneous heat-conduction equation

$$\nabla^2T - \frac{1}{\beta_s}\frac{\partial T}{\partial t} = -\frac{1}{2k_s}\tilde{Q}^{TE}e^{i\Omega t}e^{-2k_{z_2}\cdot z}, \tag{20}$$

the right-hand side of which includes energy dissipation for the TE-mode of Bessel light beam (Q is modulation frequency).

Furthermore, we take into account the dependence of the thermoelastic coupling coefficient on the initial strain for the mechanically strained crystal. Based on the results of [15, 10], where the expression for the strains with allowance for nonlinear displacements was reported, we can write the following equation for elastic displacements (6).

Having jointly solved Eqs. (19) and (6), one can easily obtain the expression for the displacement of particles in the sample under study caused by strains induced by laser radiation modulated at the frequency Ω:

$$\Delta U_3^{TE}(z) = A_1e^{-iQz} + A_2e^{iQz} + \bar{E}^{TE}\left(\frac{e^{-\beta z}}{\beta^2 + k^2}\right) - \left(\frac{e^{-\sigma_s z}}{\sigma_2^z + k^z}\right), \tag{21}$$

following designations: $\bar{E}^{TE} = \frac{g^{(3)}\beta E^{TE}}{G_3^{(3)}}$, $Q = \sqrt{\frac{\rho_0\Omega^2}{G_3^{(3)}}}$, $E^{TE} = \frac{\tilde{Q}^{TE}}{2k_s(\beta^2 - \sigma_s^2)}$, $\beta = \frac{4\pi}{\lambda}\frac{\varepsilon_2}{\sqrt{\varepsilon_1}}$ – light absorption coefficient, $k = \frac{\Omega}{V_s}$ – is the wavenumber of the sound wave in the sample, V_s is the speed of sound in the crystal, $\sigma = (1 + i)a$, $a = \sqrt{\frac{\Omega}{2\beta_s}}$ – is the thermal diffusivity.

Basing on the technique for determining the open-circuit voltage across the piezoelectric transducer [12] for a sample with free boundaries, we obtain the expression for the photoacoustic signal formed in an elastically strained gyrotropic sample:

$$V = h \cdot R \cdot W, \tag{22}$$

where the factor

$$R = \frac{\sin^2 k_1\ell_1 + 1}{\left(Qc^T\sin k_1\ell_1 + k_1c^Dctg Q\ell\cos k_1\ell_1\right)\left(\sin k_1\ell_1 + 1\right)}, \tag{23}$$

describes the purely acoustic properties of the crystalline sample–piezoelectric transducer system and the factor

$$W = \left[c^T Q\Psi_2 + ctgQ\ell\Psi_3 + (i\cos kQ\ell + \sin Q\ell)\left(ctgQ\frac{k_1 c^D}{Qc^T} - 1\right)\left(c^T\Psi_1 + \varphi\right)\right], \quad (24)$$

is responsible for the thermophysical, gyrotropic, dichroic, and thermoelastic properties of the inhomogeneous sample with internal stress. In equations (3.10 – 3.12),

$$\Psi_1 = \bar{E}^{TE}\left(\frac{\sigma_s}{\sigma_s^2+Q^2} - \frac{\beta}{\beta^2+Q^2}\right), \quad \Psi_2 = \bar{E}^{TE}\left(\frac{e^{-\beta\ell}}{\beta^2+Q^2} - \frac{\beta}{\sigma_s^2+Q_2}\right), \quad (25)$$

$$\Psi_2 = \bar{E}^{TE}\left(\frac{\sigma_s e^{-\sigma_s\ell}}{\sigma_s^2+Q^2} - \frac{\beta e^{-\sigma_s\ell}}{\beta^2+Q_2}\right) - B\alpha_t E^{TE}\left(\frac{\beta}{\sigma_s}e^{-\sigma_s\ell} - e^{-\beta\ell}\right),$$

$$\varphi_1 = B\alpha_t E^{TE}\frac{\beta-\sigma_s}{\sigma_s}, \quad (26)$$

$h = e/\varepsilon^s$, e – is the piezoelectric modulus, ε^s– is the permittivity of clamped crystal, $c^D = c^E\left(1+e^2\right)/\left(\varepsilon^s c^E\right)$, c^E – is the piezoelectric stiffness; $c^T = \lambda + 2\mu$, λ – is the Lame coefficient; B – is the bulk elasticity modulus; α_t – is the coefficient of thermal volume expansion, $k_1 = \frac{\Omega}{V_p}$ – is the wavenumber of the sound wave in the piezoelectric transducer is the wavenumber of the sound wave in the piezoelectric transducer, and V_p – is the speed of sound in the piezoelectric crystal.

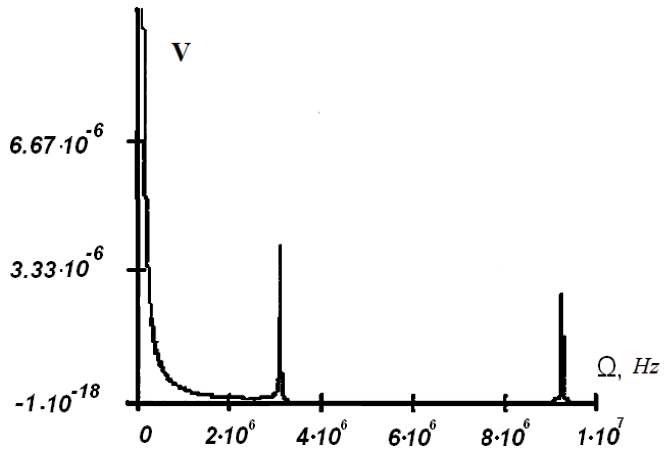

Fig. 5. Dependence of the photoacoustic signal amplitude on the modulation frequency Ω

It follows from relations (3.10–3.12) that the amplitude signal from the piezoelectric cell depends in a complicated way on the dissipative, gyrotropic, and thermophysical properties of the sample; the thermoelastic strains; the geometric parameters of the sample–piezoelectric transducer system; the modulation frequency; and the mode composition of the incident Bessel light beam.

The expressions for the potential differences arising in the piezoelectric transducer under other boundary conditions – clamped $U(o) = 0$, $U(\ell+\ell_1) = 0$ and transversely loaded $\sigma(o) = 0$, $U(\ell+\ell_2) = 0$, $\sigma(\ell+\ell_2) = 0$, $U(0) = 0$ boundaries of the crystalline sample–piezoelectric transducer system, were not considered.

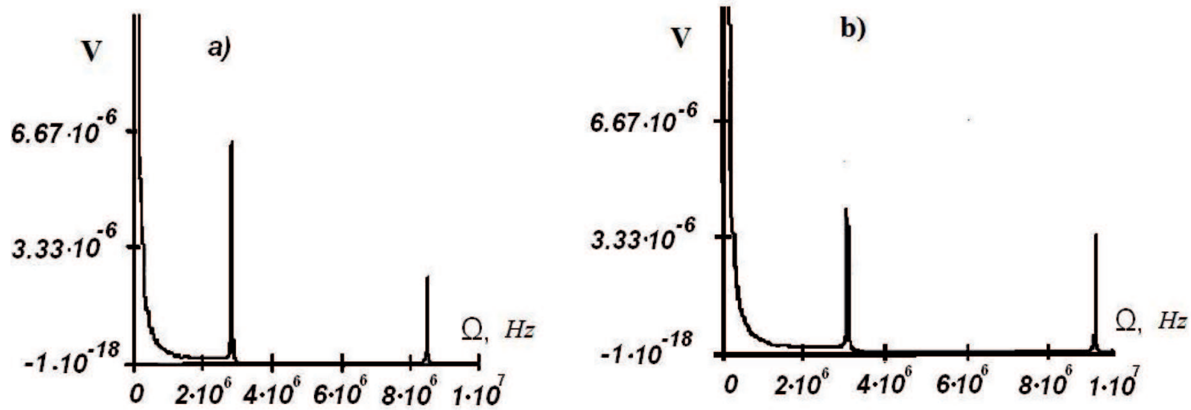

Fig. 6. Dependences of the photoacoustic signal amplitude on the modulation frequency Ω at different geometric parameters of the sample–piezoelectric transducer system: a) $\ell = 5,5\cdot10^{-3}m$, for $\ell = const$ b) $\ell_1 = 5,5\cdot10^{-4}m$, for $\ell = const$

It is of interest to analyze the expressions (3.10–3.12) for the photoacoustic signal amplitude depending on the modulation frequency Ω.

As follows from Fig. 5, in the range of high modulation frequencies (Ω > 1 MHz), the dependence of the photoacoustic response amplitude on the radial coordinate ρ exhibits resonant phenomena, which can be used to increase the resolution of photoacoustic spectroscopy for media with internal stresses.

From the acoustical point of view, the nature of the resonant phenomena is as follows. The system under consideration (crystalline sample–piezoelectric transducer) is a combined vibrator. Its resonant properties are determined by the R poles, which are roots of Eq. (23):

$$Qc^T \sin k_1\ell_1 = -k_1 c^D ctgQ\ell \cos k_1\ell_1. \quad (27)$$

Therefore, depending on the relations between the geometric parameters of the sample and piezoelectric transducer and on the Murnagan constants, which enter Q and determine the thermoelastic strain in the crystal, resonances of different types (half-wave, quarter-wave, and of a mixed type) may occur.

It can be seen in Fig. 6 that a change in the thickness of the sample or piezoelectric transducer leads to an increase in the amplitude signal and a shift in the resonance curves in the frequency range under consideration.

The expressions for amplitudes of photoacoustic signals in strained crystalline samples were obtained under different boundary conditions, taking into account the dependence of the thermoelastic coupling coefficient on the initial strain in the sample. In the range of high modulation frequencies, a resonant increase in the amplitude signal was revealed; this increment depends strongly on the geometric parameters of the sample–piezoelectric transducer system, the values of Murnagan constants, the mode composition of the Bessel light beam, and its amplitude modulation frequency.

Note that the experimental measurement of the photoacoustic response amplitude suggests a method for determining the thermoelastic coupling coefficient in crystalline media with internal stress based on the obtained expressions (11), (21) – (23).

4. Concluding remarks

It should be noted that when considering the photoacoustic transformation in gyrotropic in piezoelectric materials with internal stresses, it can be used an expression of the speed of dissipation of energy, which relates the expression (3) in work [15]. Then the temperature field distribution for TE-mode in piezoceramic materials can be presented in the form:

$$T^{TE}(z)=U_0 exp(-\sigma z)-\frac{Q_+^{TE}}{\alpha_+^2-\beta_s^2}exp(-\alpha_+ z)-\frac{Q_-^{TE}}{\alpha_-^2-\beta_s^2}exp(-\alpha_+ z)\ , \quad (4.1)$$

where: $\alpha_\pm = \frac{4\pi}{\lambda}((\frac{\varepsilon''}{2\sqrt{\varepsilon'}}) \pm \gamma'')$, $\sigma = (1+i)a$, $a = \sqrt{\frac{\Omega}{2\beta_i}}$ – the thermal diffusivity coefficient.

The combined solution (4.1) with the equation for elastic displacement (6), containing a Murnagan constant, corresponding to nonlinear termoelasticity, will determine the stress field in the sample. Taking into account that in a given solution, piezoelectric sample is simultaneously piezodetector, basing on the type of Dyugamel-Neumann equation and relationships for elastic fields, one can get an expression for the potential difference in piezoceramic materials with elastic stresses:

$$\Delta V^{TE} = V_+^{TE} - V_-^{TE},$$
$$V_\pm^{TE} = -\int_0^l E_{3\pm}^{TE}(z)dz\ . \quad (28)$$

The particular type of expression for V, and their analysis is proposed to be carried out in a separate work.

Acknowledgements

This study was supported by the Belarussian Republican Foundation for Basic Research (project No F10 R243).

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Received: 20 March 2011.