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ON THE EFFECT OF A VARIABLE THERMAL CONTACT RESISTANCE ON THE SOLIDIFICATION PROCESS

WPŁYW ZMIENNEGO CIEPLNEGO OPORU KONTAKTU NA PROCES KRZEPNIĘCIA

In this paper, the influence of a variable thermal contact resistance between a cold plate and a flowing liquid on the solidification process is theoretically analysed. A contact layer which exists between the crust and the cold plate, causes additional resistance for the flowing heat. The structure of this layer is complex. A non-pure substance occupies this volume. Gaps of air can also be present. A non-linear differential equation for the behaviour of the thickness of the solidification front is derived and solved analytically and numerically. In the contact layer there exists temperature difference between the temperature of the frozen layer plane of the opposite cold plate and the temperature of the wall of the cold plate. The thermal contact resistance depends strongly on solidification time. The general shape of this function has been prescribed by using results from other experimental studies.

W pracy określono teoretycznie wpływ zmiennego cieplnego oporu kontaktu między zimną płytą i przepływającą cieczą na proces krzepnięcia. Warstwa kontaktu powstająca między warstwą zakrzepłą a zimną płytą tworzy dodatkowy opór cieplny. Struktura warstwy kontaktu jest złożona, w objętości której mogą występować zanieczyszczenia i pęcherzyki powietrza. Sformułowano i rozwiązano analitycznie i numerycznie nieliniowe równanie różniczkowe opisuje rozwój grubości warstwy frontu krzepnięcia. W warstwie kontaktu występuje różnica temperatur między temperaturą powierzchni warstwy zakrzepłej i temperaturą powierzchni zimnej płyty. Opór cieplny kontaktu, którego funkcja opisana jest na podstawie danych eksperymentalnych zależy silnie od czasu krzepnięcia.

List of symbols

λ_l, c_l, ρ_l — thermal conductivity, specific heat and density of liquid metal;
λ_c, c_c, ρ_c — thermal conductivity, specific heat and density of cold plate;
β — constant;

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L	—	latent heat of liquid metal fusion;
H	—	cold plate thickness;
T_F	—	liquid metal fusing point;
T_∞	—	temperature of liquid metal far from frozen surface layer;
T_0	—	initial temperature of cold plate;
α	—	heat transfer coefficient between liquid metal and crust;
α_{CON}	—	heat transfer coefficient between liquid metal-cold plate contact layer;
T_W	—	temperature in plane of cold plate;
δ_1	—	frozen layer thickness;
t	—	time

1. Introduction

The paper examines analytically the influence of the variable thermal contact resistance between a cold plate and a flowing liquid on the solidification process. These phenomena play very important role in manufacturing of printed circuit electronics and also in the casting of metals. The present study focuses on the prediction of the time dependent formation of the frozen layer thickness on the cold plate.

The subject of solidification of flowing liquid on a cold plane wall has already been examined by many authors in the works [1-11]. These papers concern the cases of an isothermal cold plane wall and a cold wall of negligible heat capacity. The problem of the unsteady behaviour of the frozen layer in the liquid flowing over the cold plane wall was studied analytically by Epstein [3]. Ignaszak et al [4] examined the solidification of a flowing liquid on the cold plane wall with the contact layer between the cold plane and frozen layer. Using a very simple theoretical model they assumed a constant temperature in the area of the cold plate. The temperature was only dependent on time. Lipnicki [11] analyzed, the role of the contact layer between metal and solid in the solidification process for the constant resistance. He described behaviour of the frozen layer, which is created on the cold plane.

It is known from experiments [7-10, 12] that the resistance of the contact layer between the cold plate and the frozen layer changes with time. The thickness of the contact layer also changes with time. The layer can be considered as a solid solution of one metal in another. with complex structure which differs significantly from adjacent material.

Overall view of the contact layer between both the cold plate (copper plate) and the frozen layer (tin crust), which was formed by the solidification process, is shown in Figs. 1 and 2. Most of the impurities and defects like air gaps are here located. There is an additional resistance of the flow of heat from the heater place to the cooler place.

This paper is focused on the case when the resistance and the conductivity of the contact layer in the beginning of the solidification process are strongly time dependent. The shape of the analytical function of the resistance, which approximates the experimental data [5-10, 12], will be presented. This makes possible to provide analytical solution for the problem of the solidification of the liquid metal on the cold plane.

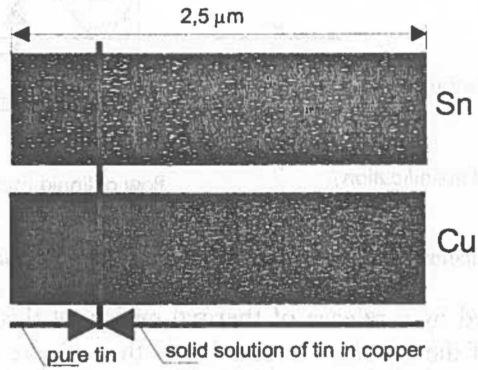

Fig. 1. Mapping line scan images of Sn and Cu of the contact layer [11]

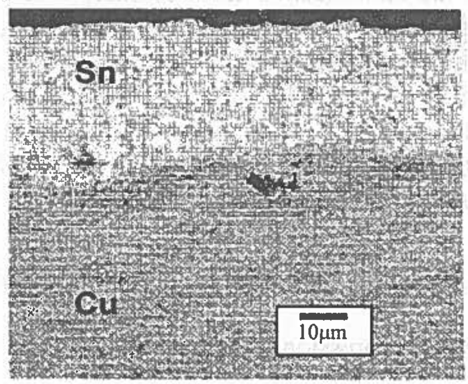

Fig. 2. Scanning electron microscopy micrograph of the contact layer

2. Formulation of the problem

It is assumed that the flowing liquid metal (see Fig. 3) is not overheated and it has the temperature T_F, which is equal to the fusion temperature of the material. The frozen layer with thickness δ_1 is formed on the plane. The temperature of cooling plane of the cold plate is equal to T_W and depends only on time t. There is a contact layer between the crust and the copper plate which causes additional variable resistance of the flowing heat. In the contact layer, there is temperature difference $\Delta T = \overline{T}_W - T_W$, where $\overline{T}_W$ is the temperature of the plane of the frozen layer of the opposite cold plate. The other plane of the cold plate is adiabatic.

As a result of the solidification process, the solidification interface grows from the cold plate with the velocity $d\delta_1/dt$. The liquid-solid phase change of the heat transfer

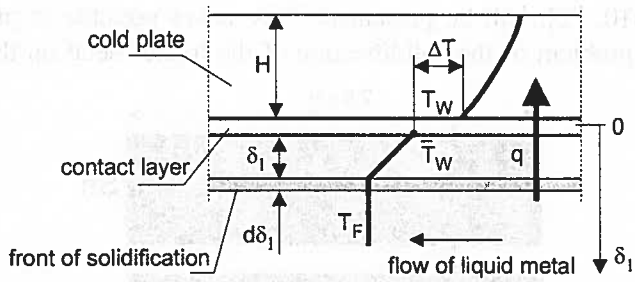

Fig. 3. Schematic representation of the contact layer indicating all the parameters used in calculations

process is accompanied by a release of thermal energy at this place. The heat flux q released at the front of the interface is transferred through the frozen layer to the cold plate. The cold plate absorbs all the heat released and it is cooled from outside with a known intensity.

It is assumed that the solidification front is sharp and planar. Furthermore, the change of the accumulation heat of the frozen layer is very small in comparison to that of the cold plate. All material properties are considered to be constant.

The problem shown in Fig. 3 can be solved applying an energy balance by comparing the instantaneous latent heat of fusion and convection heat from the flowing liquid metal with the conductive flux through the frozen layer, and with the heat through the contact layer. This results in the following equation:

$$\rho_l L \frac{d\delta_1}{dt} + \alpha\left(T_\infty - T_F\right) = \lambda_l \frac{T_F - \overline{T}_W}{\delta_1} = \alpha_{CON}\left(\overline{T} - T_W\right). \qquad (1)$$

From Eq. (1) the following expression is obtained.

$$\overline{T}_W = T_W + \rho_s \frac{l}{\alpha_{CON}} \frac{d\delta_1}{dt} + (T_\infty - T_F)\frac{\alpha}{\alpha_{CON}}.$$

For further calculations the following set of dimensionless parameters is introduced:

$$\tau = Fo \cdot Ste,\ \delta = \frac{\delta_1}{H},\ \theta = \frac{T - T_0}{T_F - T_0},\ \theta_W = \frac{T_W - T_0}{T_F - T_0},$$

where τ is a dimensionless time, δ is the dimensionless thickness of the crust layer and θ, θ_W are dimensionless temperature ratios. Furthermore, the Fourier, the Stefan and the Biot number is defined in the form of following equations

$$Fo = \frac{t\, a_l}{H^2};\ Ste = \frac{c_l\left(T_F - T_0\right)}{L};\ Bi = \frac{H\alpha_{CON}}{\lambda_l},$$

and for a and λ the following abbreviations:

$$a = \frac{a_C}{a_l};\ \lambda = \frac{\lambda_C}{\lambda_l}.$$

In view of the assumption that the liquid flowing metal is not overheated, the heat does not flow to the cold plate by convection. Inserting dimensionless parameters defined above, it is possible to re-write eq.(1) in the form:

$$1-\theta_W=\frac{\delta d\delta}{d\tau}+\frac{1}{\mathrm{Bi}}\frac{d\delta}{d\tau}. \tag{2}$$

In order to solve the differential equation (2), the wall temperature $\theta_W(\tau)$ has to be known. In addition, it is necessary to define the boundary condition between the plate and the crust liquid.

3. Solution of the problem

The wall temperature on the plane of the cold plate is assumed as:

$$\theta_W=1-\exp(-\beta\tau) \tag{3}$$

with the constant β, which will be determined by the outside conditions.
The shape of this function is in a good agreement with the real conditions of cooling, which occur in the solidification process. Intensity of cooling of the contact layer depends on the value of the parameter β. The way of determining the constant β was given in paper [11]. From equations (2) and (3) the differential equation which describes the solidification process on the cold plate is obtained:

$$\frac{1}{2}\frac{d\delta^2}{d\tau}+\frac{1}{Bi(\tau)}\frac{d\delta}{d\tau}=\exp(-\beta\tau). \tag{4}$$

One can note that Biot number, which determines here properties of the contact layer varies with time. A theoretical form of this function is needed to solve equation (4). The shape of the Biot number can be received from the experimental data of Wang and Mattheys [12]: At the beginning of the solidification process, a very fast increase of the contact conductive value (also the Biot number) from zero to its maximum value takes place. Then, this Biot number decreases very fast and attains a steady-state constant value. Figure 4 shows the distribution of the Biot number as a function of time.
The Biot number distribution depicted in Fig. 4 can be approximated by:

$$Bi(\tau)=Bi_0\sin\left(0.85\frac{\pi\tau}{\tau_{ste}}\right) \qquad \text{for} \quad 0\langle\tau\langle\tau_{ste} \tag{5}$$

$$Bi(\tau)=Bi_{ste} \qquad \text{for} \quad \tau_{ste}\langle\tau\langle\infty, \tag{6}$$

where: both Bi_0 and Bi_{ste} are the maximum and the steady-state Biot numbers, τ_{ste} is the time where the steady-state heat resistance of the contact layer is achieved.

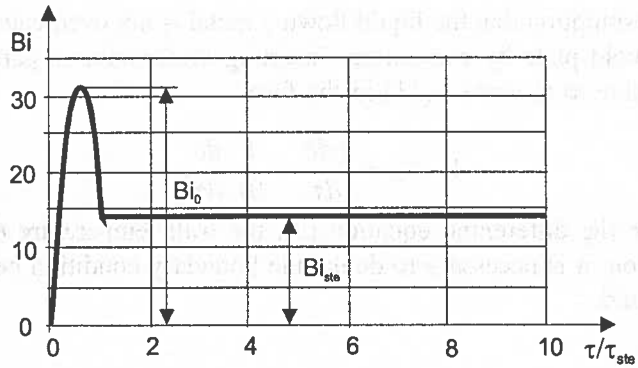

Fig. 4. Dependence of the Biot number on the relative time in the contact layer based on experimental work of Wang and Matthyes [12]

At the beginning of the solidification process, we can assume that the frozen layer thickness is very small, $\delta\langle\langle 1$, and we neglect the quadratic term in equation (4) which results in:

$$\frac{1}{Bi(\tau)}\frac{d\delta}{d\tau} = \exp(-\beta\tau). \tag{7}$$

For short time of solidification the Biot number can be approximated by:

$$\mathrm{Bi}(\tau) = \mathrm{Bi}_0 \sin\left(0.85\,\pi\frac{\tau}{\tau_{ste}}\right). \tag{8}$$

Inserting equation (8) into equation (7) leads to:

$$\frac{1}{\mathrm{Bi}_0 \sin\left(0.85\,\pi\frac{\tau}{\tau_{ste}}\right)}\frac{d\delta}{d\tau} = \exp(-\beta\tau). \tag{9}$$

For equation (9) an analytical solution can be derived by the method of separation of variables.

$$\delta = \mathrm{Bi}_0 \int_0^{\tau} \sin\left(0.85\pi\frac{\tau}{\tau_{ste}}\right)\exp(-\beta\tau)\,d\tau = \mathrm{Bi}_0\frac{\exp(-\beta\tau)}{\beta^2 + \left(0.85\frac{\pi}{\tau_{ste}}\right)^2}\left[-\beta\sin\left(0.85\,\pi\frac{\tau}{\tau_{ste}}\right) - \right.$$

$$\left. -0.85\frac{\pi}{\tau_{ste}}\cos\left(0.85\cdot\pi\frac{\tau}{\tau_{ste}}\right)\right]_0^{\tau}.$$

Carrying out the integration, one can obtain:

$$\delta = \frac{\mathrm{Bi}_0}{\beta^2 + \left(0.85\frac{\pi}{\tau_{ste}}\right)^2}\left[0.85\frac{\pi}{\tau_{ste}}\left(1-\exp(-\beta\tau)\cos\left(0.85\pi\frac{\tau}{\tau_{ste}}\right)\right) - \beta\exp(-\beta\tau)\sin\left(0.85\pi\frac{\tau}{\tau_{ste}}\right)\right]. \tag{10}$$

If the B i o t number can be approximated by a constant value, the frozen layer is given by

$$\delta = -\frac{1}{Bi_{ste}} \sqrt{\frac{1}{Bi_{ste}^2} + \frac{2}{\beta}\left(1 - \exp\left(-\beta\tau_{ste}\frac{\tau}{\tau_{ste}}\right)\right)}. \tag{11}$$

The equation (4) also was solved numerically by a R u n g e - K u t t a method [13, 14].

4. Results

The obtained results are presented in Figs. 5 – 9. Figures 5 and 6 show the theoretical distribution of the frozen layer for three different cooling conditions using two analytical solutions of the problem given by Eq. 7 and Eq. 11, respectively.

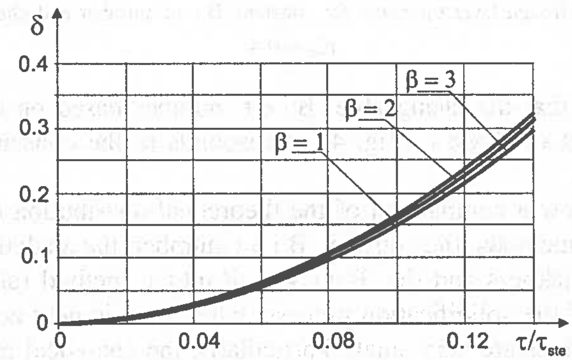

Fig. 5. Frozen layer thickness for changeable B i o t number, $\tau_{ste} = 0.4$

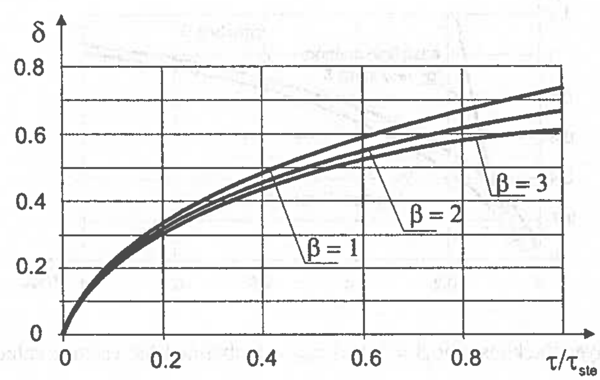

Fig. 6. Frozen layer thickness for constant B i o t number

Figure 7 shows a comparison of the predicted theoretical distribution of the frozen layer thickness for both the constant and the time dependent Biot number (Eq. 7).

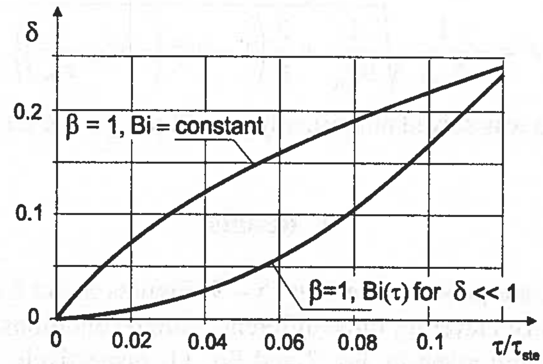

Fig. 7. Comparison of frozen layer thickness for constant Biot number and changeable Biot number, $\tau_{ste} = 0.4$

One can conclude that the changeable Biot number based on experimental work of Wang and Matthyes (Fig. 4) corresponds to the constant Biot number averaged over time

Figures 8 and 9 show a comparison of the theoretical distribution of the frozen layer thickness for three methods: the constant Biot number, the analytical method for the very small layer thickness and the Runge-Kutta method (after Ref. [13, 14]). In the beginning of the solidification process, when time is near zero, the differences between those methods are very small. Particularly, the analytical methods for $\delta \langle\langle 1$ in comparison with the Runge-Kutta numerical method lead to similar results for the short times ($\tau/\tau_{ste} \rightarrow 0$). The increase of the layer thickness with time is also similar

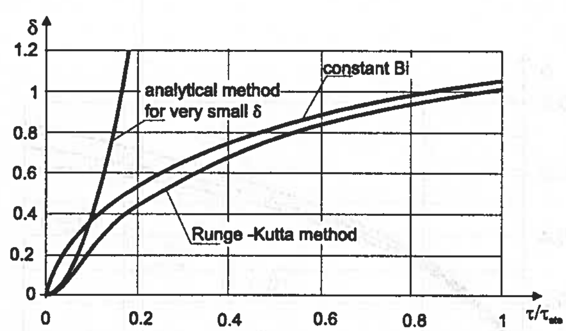

Fig. 8. Frozen layer thickness for $\beta = 1$ and $\tau_{ste} = 1$ obtained for various calculation methods

(a curve is concave) in contrast to the increase of the layer thickness in the constant Biot number approximation (a curve is convex). For very large times ($\tau/\tau_{ste} \rightarrow 1$) the constant Biot number approximation provides a fair approximation of real conditions.

The analytical method for a very small parameter δ is very useful, because it allows determining the frozen layer thickness for very short times ($\tau/\tau_{ste} \rightarrow$).

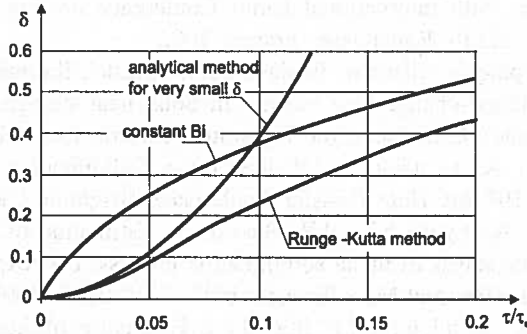

Fig. 9. Frozen layer thickness in the beginning solidification for $\beta = 1$ and $\tau_{ste} = 1$ obtained for various calculation methods

5. Concluding remarks

The development of the frozen layer in liquid metal depends on both the way of chilling and the thermal contact resistance between the cold plate and the frozen layer.

In the beginning when the thermal resistance (Biot number) is small, the increase of frozen layer is slow. Then the strong increase of the frozen layer with time is observd. Development of the frozen layer is more realistic for the Biot number dependent on time than for the constant Biot number. The variable thermal contact resistance influences evidently the solidification phenomena.

From the above analysis, we can draw the following conclusions:

— there is a need to show the relationship between the macrostructure and the microstructure with the aim to find a macro parameters which will describe the contact layer,

— there is a need to create the couple of theoretical models which will describe both the diffusivity of heat and diffusivity of mass in the solidification process in the contact layer.

REFERENCES

[1] C. Lapadula, W.K. Mueller, Heat conduction with solidification and convection boundary condition at the freezing front, Int. J. Heat Mass Transfer **9**, 702- 705, 1966.

[2] J.M. Saviano, R. Siegel, An analitical solution for solidification of a moving warm liquid onto an isothermal cold wall, Int. J. Heat Mass Transfer **12**, 803- 809 (1969).

[3] M. E p s t e i n, The growth and decay of a frozen layer in forced flow, Int. J. Heat Mass Transfer **19**, 1281-1288 (1976).
[4] Z. I g n a s z a k, Z. L i p n i c k i, A. B y d a ł e k, Solidification of liquid metal flowing below a cold plate, Fifth International Pamir Conference on Fundamental and Applied MHD, vol. 2, II131-II136, Ramatuelle, France, 2002
[5] W. L o n g a, Krzepnięcie odlewów, Wydawnictwo „Śląsk", Katowice, 1985.
[6] R. V i s k a n t a, Phase-change heat transfer in Solar heat storage, Latent Heat Transfer (Edited by G.A. Lane), CRS Press, Int. Bocaratod, Florida 153-221 (1983).
[7] T. L o u l o u, E.A. A r t y u k h i n, J.P. B a r d o n, Solidification of molten tin drop on a nickel substrate, 10^{th} Int. Heat Transfer Conference, Brighton, UK **4**, 73-78 (1998).
[8] T. L o u l o u, J.P. A r t y u k h i n, J.P. B a r d o n, Estimation of thermal contact resistance during the first stages of metal solidification process: I — experiment principle and modelisation, Int. J. Heat and Mass Transfer bf42, 2119-2127 (1999).
[9] T. SoLoulou, J.P. A r t y u k h i n, J.P. B a r d o n, Estimation of thermal contact resistance during the first stages of metal solidification process: II — experimental setup and results , Int. J. Heat and Mass Transfer 42, 2129-2142 (1999).
[10] P. F u r m a ń s k i, T.S. W i ś n i e w s k i, Thermal contact resistance and thermal phenomena at solid-solid interface, Institute of Heat Engineering, Warsaw University of Technology, Warsaw 2002.
[11] Z. L i p n i c k i, Role of the contact layer between liquid and solid on solidification process, Int. J. Heat and Mass Transfer 46, 2149-2154 (2003).
[12] G.X. W a n g, E.F. M a t t h y e s, Interfacial thermal contact during rapid solidification on substrate, Proceedings of 10^{th} International HeatTransfer Conference, Brighton, UK **4**, 169-174 (1994).
[13] M. A b r a m o w i t z, I.A. S t e g n, Handbook of mathematical functions, National Bureau of Standards Applied Mathematics Series 55, 1964.
[14] Z. F o r t u n a, B. M a c u k o w, J. W ą s o w s k i Metody numeryczne, WNT, Warszawa, 2001.

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