A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 53 2008 Issue 3

K. WIENCEK*

WEIBULL DISTRIBUTION IN QUANTITATIVE METALLOGRAPHY OF PARTICLE SYSTEMS

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Stereology for spheres, whose diameters distribution corresponds to the Weibull one is presented. Estimation method for parameters of the Weibull probability density function (PDF) is proposed. In some metalic materials the particles (disperse carbide phase, grains in polycrystalline metals) may be assumed as spheres whose diameter distributions may be described by the Weibull PDF. Consequently, the measurement of particle size distribution may be reduced to stereological estimation of the Weibull PDF parameters.

Przedstawiono stereologię dla kul, których średnice mają rozkład Weibulla. Zaproponowano metodę estymacji parametrów funkcji gęstości prawdopodobieństwa (PDF) rozkładu Weibulla. W niektórych materiałach metalicznych cząstki (dyspersyjne fazy węglikowe, ziarna polikrystalicznych metali) można aproksymować kulami, których średnice mają rozkład Weibulla. W konsekwencji, pomiar rozkładu rozmiarów cząstek sprowadza się do stereologicznej estymacji parametrów funkcji PDF rozkładu Weibulla.

1. Introduction

A phase in material microstructure may be formed of isolated particles arranged statistically uniform in the space (e.g. disperse phase, grains of recrystallized metal, etc.). The fundamental geometric characteristics of a particle is the size, i.e., one dimensional measure of the largeness. Statistically, particle size is interpreted as a value of continue random variable (random size) whose probability distribution is determined by the probability density function (PDF). The PDF describes the distribution type (e.g., the distributions like logarithmic normal, gamma, Weibull, etc.). In quantitative metallography, the probability distribution of random size is known as the particle size distribution function [1]. Empirical studies of the size distribution are based on measurement of the PDF by stereological methods. The selection of measurement method depends on geometric properties of the particles, the type of the size distribution and the required precision. In a first approximation particles are regarded as spheres. If the distribution type is not known, for measurement usually the so-called Saltykow algorithm or its later versions [1-4] are used. Deficiences of the Saltykow algorithm limit its applications [5]. However, if the distribution type is known, the measurement of the PDF may by reduced to its parameters only (the so-called parametric method [5]). Of practical meanings are usually two parameter functions only [5]. Some metallographic studies of: carbide dispersions in steels, graphite in nodular cast iron, and grains in recrystallized metals and one phase alloys indicate the distributions: logarithmic normal [2, 6], Rayleigh [7-9] and Weibull [10]. Saltykow and DeHoff formulated a stereology for spheres with logarithmic normally distributed diameters [2, 11]. Ryś and Wiencek have analysed similar problem for the Rayleigh distribution [12]. The Rayleigh distribution is a special case of the Weibull distribution, however a suitable stereology for the latter does not exist yet. The aim of this work is extending the stereology of spheres by taking into consideration the Weibull diameter distribution case. The contents includes: (i) main ideas of the general stereology for spheres; (ii) outlines of stereology for spheres with Weibull diameter distribution; (iii) analysis of empirical size distributions from point of view of the Weibull distribution. The stereological considerations are limited to statistical quantities only (PDF, moments, etc.).

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2. General stereology of spheres

A random set of non-overlapping spheres is given (spheres of random diameter D). The sphere diameter D is a random variable of PDF $f_3(D)$, whose values belong to the interval $[0, \infty]$. The mean

$$\langle D^r \rangle = \int_0^\infty D^r f_3(D)\, dD \quad (r = 0, 1, 2, ...), \tag{1}$$

is the r order sphere diameter moment. For $r = 1$, $\langle D \rangle$ is the simple expected value. For $r = 2$, the mean square $\langle D^2 \rangle$ and $\langle D \rangle$ define the variance $\sigma_D^2 = \langle D^2 \rangle - \langle D \rangle^2$ and the variation coefficient $v_D = \sigma_D / \langle D \rangle$, as well.
A circle (profile) is a planar section of sphere. The section of a sphere system is a random set of profiles in the plane. The profile diameter d is a random variable of PDF $f_2(d)$. The mean

$$\langle d^r \rangle = \int_0^\infty d^r f_2(d)\, dd \quad (r = -1, 0, 1, ...), \tag{2}$$

is the r order profile diameter moment.
The diameter moments, $\langle D^r \rangle$ and $\langle d^r \rangle$, satisfy the equation

$$\langle d^r \rangle = \frac{\sqrt{\pi}\,\Gamma\left(\frac{r+2}{2}\right)}{2\,\Gamma\left(\frac{r+3}{2}\right)} \left\langle D^{r+1} \right\rangle \langle D \rangle^{-1} \quad (r = -1, 0, 1, ...), \tag{3}$$

where Γ is Euler gamma function [13]. In particular, from Eq. (3) for $r = 1$, it follows

$$\frac{\langle d \rangle}{\langle D \rangle} = \frac{\pi}{4}\left(1 + v_D^2\right). \tag{4}$$

For equal spheres ($\langle D \rangle = D$ and $\sigma_D = 0$), Eq. (4) shows that $\langle d \rangle = (\pi/4)D$. This means $\langle d \rangle / \langle D \rangle \geqslant (\pi/4)$, then for $v_D > 0.523$, $\langle d \rangle / \langle D \rangle$ is greater than one which seems paradoxical. The PDFs: $f_3(D)$ and $f_2(d)$ satisfy the Wicksell equation [5, 13]:

$$f_2(d) = \langle D \rangle^{-1} d \int_d^\infty \frac{f_3(D)}{\sqrt{D^2 - d^2}} dD \quad (d \geqslant 0). \tag{5}$$

For a given $f_2(d)$, Eq. (5) is an integral equation which has analytical solution with respect to the unknown $f_3(D)$.

For real sphere systems, the diameter D belongs to the interval $[0, D_m]$, where D_m is the largest diameter. In such cases, the solution of Eq. (5) may be obtained by numerical methods only. The most of numerical methods for solution of the integral equation Eq. (5) result from the Saltykow algorithm [3].

2.1. Saltykow algorithm

The $[0, D_m]$ diameter interval is divided into k equal intervals (classes) of Δ widths. The discrete profile and sphere diameters are chosen as follows: $d_i = (i - 1/2)\Delta$ and $D_j = j\Delta$, $(i, j = 1, ..., k)$. The discrete PDFs $f_2(i) = f_2(d_i)$ and $f_3(j) = f_3(D_j)$ follow the Saltykow's equation system [2]:

$$f_2(i) = \langle D \rangle^{-1} \Delta \sum_{j=i}^{k} k_{ij} f_3(j), \quad (I = 1, ..., K); \tag{6}$$

with

$$k_{ij} = \sqrt{j^2 - (i-1)^2} - \sqrt{j^2 - i^2}. \tag{7}$$

Eq. (6) and formula (3) for $r = 1$ are the basis of the Saltykow algorithm. For exact $f_2(i)$ data, the main properties of the solution of Eq. (6) (i.e., the $f_3(j)$) are dependent on the number of classes k: (i) for large k ($k > 15$), solutions obtained by the Saltykow algorithm are satisfactory; (ii) for small k the solution has a high systematic error.
For $f_2(i)$ data with random errors, the Saltykow algorithm is unstable (small perturbations of the data correspond to large perturbations of the solution) [5, 14]. At the moment, there exist new algorithms for numeric solution of the Wicksell Eq. (5), which are more stable [15]. Finally, from the present work point of view it should be noted, that for a given $f_3(D)$, Eq. (6) enables calculation of the $f_2(i)$ values.

In the stereology of spheres Eqs. (3), (5) and (6) are quite general – they are independent of the diameter distribution properties – therefore they form theoretical basis for stereological measurement methods in quantitative metallography of particles which can be approximated by spheres [1, 2].

3. Weibull stereology of spheres

Weibull stereology is a stereology of spheres with Weibull diameter distribution, whose PDF is the following

$$f_3(D) = n\alpha D^{n-1} \exp(-\alpha D^n), \tag{8}$$

where α and n are positive parameters [16]. The Weibull PDF – Eq. (8) – is of positive asymmetry, wich decreases when n increases. The substitution of Eq. (8) into Eq. (1) results in

$$\langle D^r\rangle = \alpha^{-\frac{r}{n}}\Gamma\left(\frac{r}{n}+1\right) \qquad (r = 0, 1, 2, ...). \tag{9}$$

By means of Eq. (9) for given n and $r = 1$, the α parameter can be expressed by the mean diameter $\langle D\rangle$,

$$\alpha = \left(\frac{\Gamma\left(\frac{1}{n}\right)}{n\langle D\rangle}\right)^n. \tag{10}$$

Taking Eq. (10) into consideration, Eq. (9) may be written

$$\langle D^r\rangle = \frac{\Gamma\left(\frac{r}{n}+1\right)}{\Gamma^r\left(\frac{1}{n}+1\right)}\langle D\rangle^r \qquad (r = 0, 1, 2, ...). \tag{11}$$

The substitution of Eq. (11) into Eq. (3) results in

$$\langle d^r\rangle = \frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{r+2}{2}\right)\Gamma\left(\frac{r+1}{n}+1\right)}{\Gamma\left(\frac{r+3}{2}\right)\Gamma^{r+1}\left(\frac{1}{n}+1\right)}\langle D\rangle^r \qquad (r = -1, 0, 1, , ,). \tag{12}$$

From Eq. (12) for $r = 1$ results

$$\frac{\langle d\rangle}{\langle D\rangle} = \frac{\pi}{4}\frac{\Gamma\left(\frac{2}{n}+1\right)}{\Gamma^2\left(\frac{1}{n}+1\right)}. \tag{13}$$

Eq. (13) is a special case of Eq. (4), i.e. for the Weibull distribution. From Eq. (13) for $n = 2$ results: $\langle d\rangle/\langle D\rangle = 1$; and for $n > 2$, $\langle d\rangle/\langle D\rangle < 1$. Substitution of Eq. (13) into Eq. (12) for $r = 2$ results in

$$\frac{\langle d^2\rangle}{\langle d\rangle^2} = \frac{8}{\pi^2}\frac{\Gamma\left(\frac{3}{n}\right)\Gamma\left(\frac{1}{n}\right)}{\Gamma^2\left(\frac{2}{n}\right)}. \tag{14}$$

Eq. (14) is a fundamental stereological equation in the Weibull stereology of spheres, it connects the n parameter for spheres and the profile diameter moments $\langle d\rangle$ and $\langle d^2\rangle$. In the Weibull stereology of spheres, the $f_3(D)$ in Eq. (8) can be used as parametric function, the parameters are n and α. So, the stereological estimation of the Weibull PDF $f_3(D)$ may be reduced to the estimation of parameters n and α using Eqs. (10), (13) and (14). Eqs. (10), (13) and (14) are the basis of an algorithm for estimation of the Weibull PDF $f_3(D)$, this algorithm will be called the Weibull stereology algorithm.

4. Experimental

The aim of the experimental studies was to describe some empirical particle size distributions by the Weibull distribution. The description is based on approximation of the empirical $f_2(d)$ function by the one, calculated by Wicksell equation Eq. (5) including the Weibull PDF $f_3(D)$ with n and α parameters given by Eqs. (10), (13), (14) for the empirical $\langle d\rangle$ and $\langle d^2\rangle$ moments. If the given data (the empirical $f_2(d)$) are approximated satisfactorily by the calculated $f_2(d)$, then the particle size distribution corresponds to the Weibull distribution and is described by the PDF $f_3(D)$.

The precision of the size distribution description depends on the accuracy of the approximation mentioned above. In the first approximation, the accuracy is determined by visual similarity of the $f_2(d)$ function graphs. A quantitative information according to quality of the description above gives comparison of the $\langle D^r\rangle$ moments which are determined by Eqs. (3) and (11).

Furthermore, if the particle size distribution corresponds to the Weibull one, the $f_3(D)$ estimated by the Weibull stereology algorithm and the Saltykow algorithm should be consistent each other.

4.1. Empirical data

From papers [17-20] five particle systems (A, B, C, D, E) – components of metallic material microstructures – were selected, in particular,

A – Fe_3C carbide dispersion in Fe-0.6%C steel (held at 700°C for 24h), [17], Fig. 1a;

B – coarse Fe_3C carbide dispersion in Fe-0.6%C steel (700°C/600h), [17], Fig. 1b;

C – M_6C carbide dispersion in SW18 high speed steel (1200°C/10h), [18], Fig. 1c;

D – recrystallized titanium, [19], Fig. 1d;

E – graphite in nodular cast iron, [20], Fig. 1e.

(For particle systems A, B, C in brackets, there are given the heat treatment conditions, i.e, temperature and time for particle coarsening process.)

Fig. 1 shows the material microstructures. The graphite particles are nearly spherical, the titanium grains are polyhedral, while the carbide particles are almost convex.

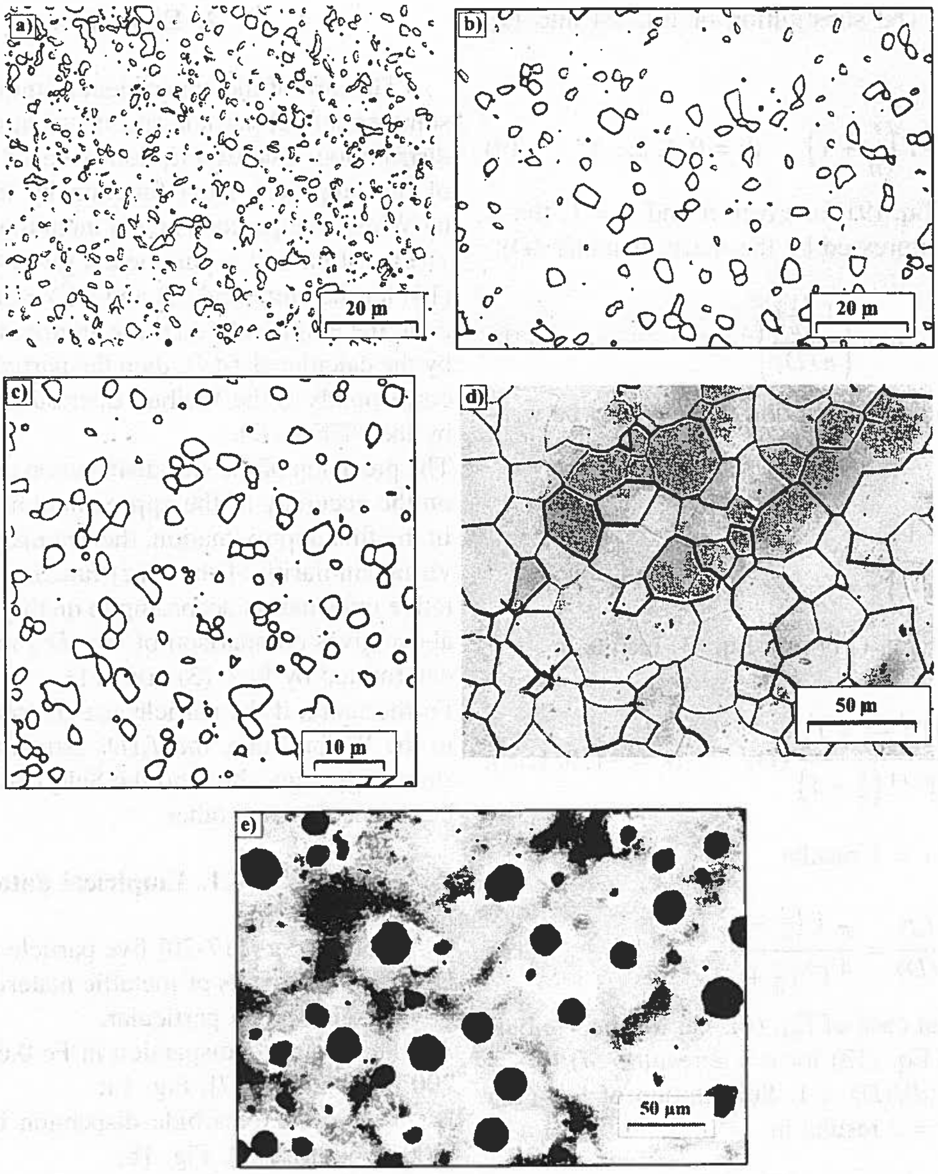

Fig. 1. Material microstructures:
(a) steel Fe-0.6%C, 700°C/25h
(b) steel Fe-0.6%C, 700°C/600h
(c) high speed steel SW18, 1200°C/10h
(d) recrystallized titanium
(e) nodular cast iron

For quantitative studies, the material microstructure particles are treated geometrically as spheres.

4.2. Quantitative metallography

The profile d diameter measurement was made by Zeiss TGZ-3 particle size analyser, which is based on the principle, that d is the diameter of a circle which area is equal to the particle area. The TGZ-3 area comparison is made visually, this may introduce uniknown errors for not spherical particles.
For the selected particle section systems with particle number N: N = 2400 (A), N = 2390 (B), N = 2300 (C), N = 550 (D) and N = 1001 (E), respectively, the profile diameters were measured and then the empirical $f_2(d)$ and moments $\langle d^r \rangle$ (r =1, 2) determined. Fig. 2 shows graphs of the empirical $f_2(d)$ functions, they are quite regular and unimodal of a positive asymmetry. The studies include:

- estimation of $f_3(D)$ by the Weibull stereology algorithm,
- analysis of $\langle D^r \rangle$ moments,
- estimation of $f_3(D)$ by the Saltykow algorithm.

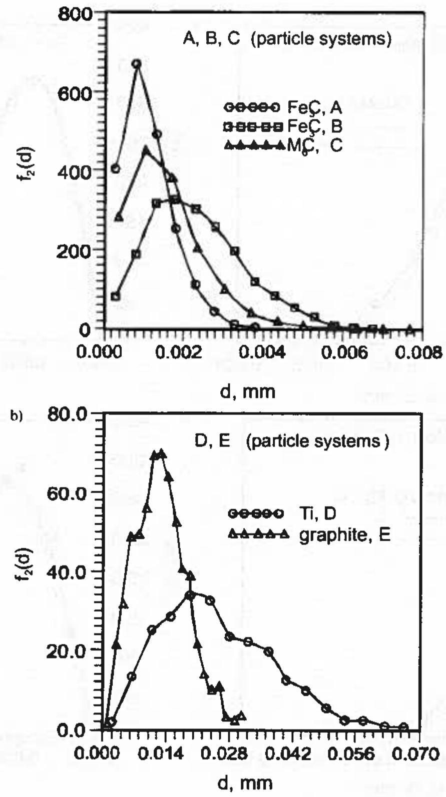

Fig. 2. Empirical $f_2(d)$ functions: (a) steels (A,B,C); (b) titanium (D) and nodular cast iron (E)

4.3. Estimation by Weibull stereology algorithm

According to Eq. (10), for a given n, the α parameter can be expressed by $\langle D \rangle$. Consequently, the n and $\langle D \rangle$ numbers were assumed as parameters of the Weibull PDF $f_3(D)$ in Eq. (8). For the empirical $f_2(d)$, the $\langle d^r \rangle$ moments (r = 1, 2) were determined. Then, by Eqs. (13) and (14) for empirical profile diameter moments ($\langle d \rangle$ and $\langle d^2 \rangle$) the n and $\langle D \rangle$ parameters were estimated, the results are presented in Table 1.

TABLE 1

Parameters of profiles and the estimated Weibull PDF $f_3(D)$ for material microstructures

particle system	$\langle d \rangle$, mm	$\langle d^2 \rangle / \langle d \rangle^2$	n	$\langle D \rangle$, mm
A	$1.05 \cdot 10^{-3}$	1.38	1.49	$0.91 \cdot 10^{-3}$
B	2.28 · 10–3	1.29	1.92	$2.24 \cdot 10^{-3}$
C	$1.53 \cdot 10^{-3}$	1.45	1.29	$1.21 \cdot 10^{-3}$
D	$2.49 \cdot 10^{-2}$	1.24	2.25	$2.61 \cdot 10^{-2}$
E	$1.25 \cdot 10^{-2}$	1.21	2.55	$1.35 \cdot 10^{-2}$

From Table 1 it results, that the n parameter values are: $1.29 \leqslant n \leqslant 2.55$ and according to Eq. (13) for $n < 2$ the $\langle d \rangle > \langle D \rangle$.

Next, by means of the $f_3(D)$ – given by Eq. (8) with the empirical n and $\langle D \rangle$ parameters (Table 1) the approximating $f_2(d)$ functions were calculated by means of Saltykow's Eq. (6). Fig. 3 shows the empirical and approximating $f_2(d)$ function graphs; a visual similarity is in principle acceptable (the largest difference is for the case C, i.e., the M_6C carbide dispersion in the high speed steel, Fig. 1c). It should be noted, that on the left of the maximum of approximating functions with largest asymmetry (n = 1.29 (C) and n = 1.49 (A)), the empirical functions are represented by one value only.

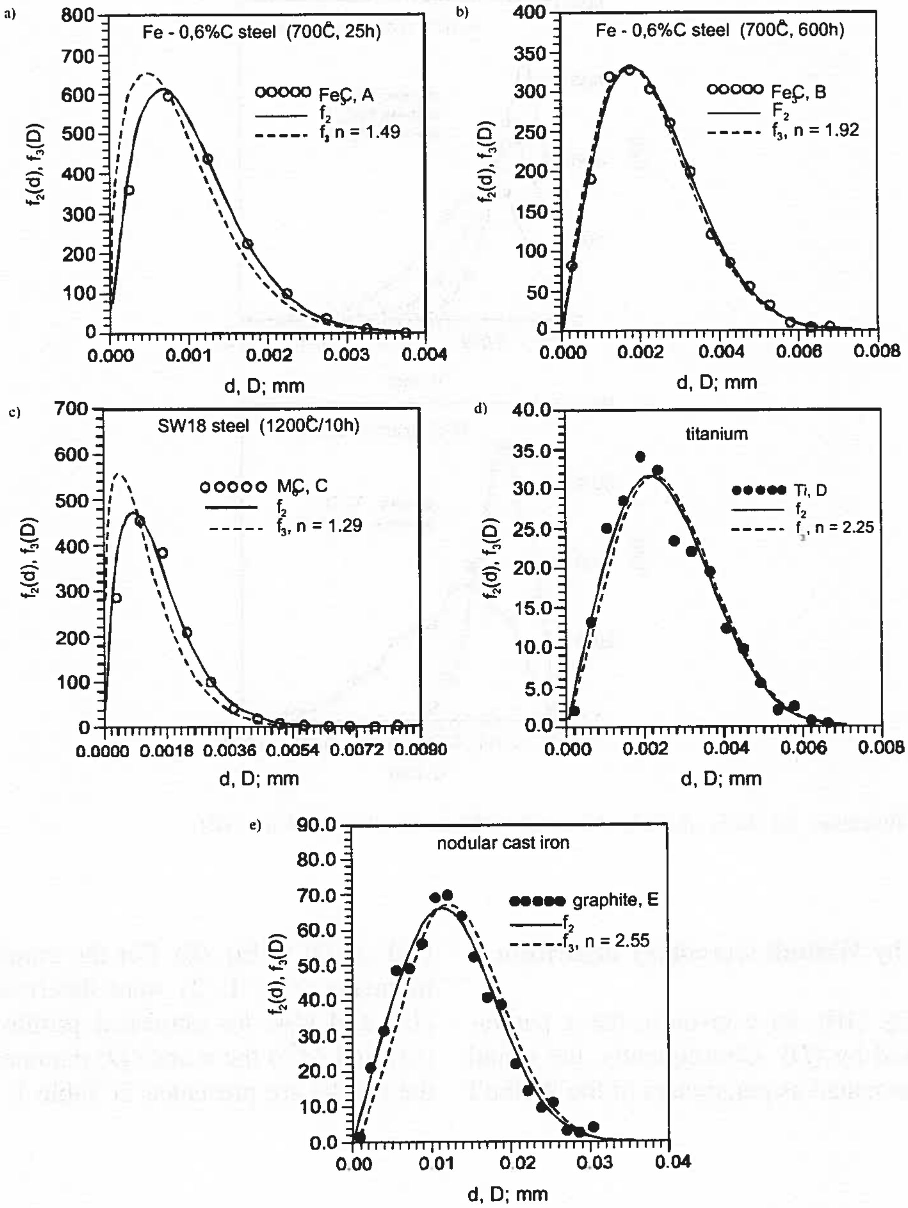

Fig. 3. Empirical $f_2(d)$ functions, its approximating functions and the Weibull PDFs $f_3(D)$ estimated by the Weibull stereology algorithm for steels (a, b, c), titanium (d) and cast iron (e)

Because, the approximating $f_2(d)$ functions fit the empirical ones, it can be concluded that for the selected particle systems the size distribution corresponds to the Weibull distribution. Fig. 3 shows also the graphs of the estimated Weibull PDF $f_3(D)$. For $n < 2$, the $f_2(d)$ graphs are shifted to the right relative to the $f_3(D)$ graphs, which is in agreement with the relation for the mean diameters: $\langle d\rangle > \langle D\rangle$, (Eq. (13) and Table 1).

4.4. Analysis of moments

The moments $\langle D^r\rangle$ ($r = 1, 2, 3$) were determined by two methods, i.e.,

I. by Eq. (3) in the general stereology of spheres, using empirical moments $\langle d^r\rangle$; and

II. by Eq. (11) for $r = 2, 3$ in the Weibull stereology of spheres using empirical parameters n and $\langle D\rangle$ (for $r = 1$ the determination of $\langle D\rangle$ is based on Eq. 13).

The results are presented in Tables 2a – 2e.

TABLE 2A

Moments $\langle D^r \rangle$ (r = 1, 2, 3) for Fe_3C dispersion (A), determined by different methods

r	$\langle D^r \rangle$	method I	method II
1	$\langle D \rangle$	$1.00 \cdot 10^{-3}$, mm	$0.91 \cdot 10^{-3}$, mm
2	$\langle D^2 \rangle$	$1.34 \cdot 10^{-6}$, mm^2	$1.22 \cdot 10^{-6}$, mm^2
3	$\langle D^3 \rangle$	$2.29 \cdot 10^{-9}$, mm^3	$2.07 \cdot 10^{-9}$, mm^3

TABLE 2B

Moments $\langle D^r \rangle$ (r = 1, 2, 3) for Fe_3C dispersion (B), determined by different methods

r	$\langle D^r \rangle$	method I	method II
1	$\langle D \rangle$	$2.26 \cdot 10^{-3}$, mm	$2.24 \cdot 10^{-3}$, mm
2	$\langle D^2 \rangle$	$6.56 \cdot 10^{-6}$, mm^2	$6.51 \cdot 10^{-6}$, mm^2
3	$\langle D^3 \rangle$	$2.27 \cdot 10^{-8}$, mm^3	$2.25 \cdot 10^{-8}$, mm^3

TABLE 2C

Moments $\langle D^r \rangle$ (r = 1, 2, 3) for M_6C dispersion (C), determined by different methods

r	$\langle D^r \rangle$	method I	method II
1	$\langle D \rangle$	$1.41 \cdot 10^{-3}$, mm	$1.21 \cdot 10^{-3}$, mm
2	$\langle D^2 \rangle$	$2.74 \cdot 10^{-6}$, mm^2	$2.35 \cdot 10^{-6}$, mm^2
3	$\langle D^3 \rangle$	$7.13 \cdot 10^{-9}$, mm^3	$6.13 \cdot 10^{-9}$, mm^3

TABLE 2D

Moments $\langle D^r \rangle$ (r = 1, 2, 3) for titanium grains (D), determined by different methods

r	$\langle D^r \rangle$	method I	method II
1	$\langle D \rangle$	$2.77 \cdot 10^{-2}$, mm	$2.61 \cdot 10^{-2}$, mm
2	$\langle D^2 \rangle$	$8.78 \cdot 10^{-4}$, mm^2	$8.30 \cdot 10^{-4}$, mm^2
3	$\langle D^3 \rangle$	$3.20 \cdot 10^{-5}$, mm^3	$3.03 \cdot 10^{-5}$, mm^3

From Tables 2a – 2e it results, that the $\langle D^r \rangle$ moment values (r = 1, 2, 3), which were determined by different methods are close each other, however the values obtained by method I are slightly higher. The relative difference of the moments (calculated relative to the method I) are less than 15%. The smallest difference (1%) corresponds to the particle system B (Fe_3C carbide dispersion in Fe-0.6%C steel coarsened for 600 hours, Fig. 1b), while the largest difference corresponds to the system C (M_6C carbide dispersion in high speed steel, Fig. 1c). It should be noted, that the largest difference between empirical and approximating $f_2(d)$ functions corresponds also to the particle system C, Fig. 3c.

4.5. Estimation by Saltykow algorithm

For the empirical $f_2(i)$ functions, i = 1, ..., k, the estimated $f_3(j)$ functions, j = 1, ..., k, were determined by the Saltykow algorithm (Eq. (6)). Fig. 4 shows the estimated $f_3(j)$ functions compared with the $f_3(D)$ functions, which is estimated by the Weibull stereology algorithm. For small j-values the differences between $f_3(j)$ and $f_3(D)$ are rather large; for the D and E particles (for j = 1, $f_3(j) < 0$) it can result from the small particle number (N), i.e., N = 550 and N = 1001, respectively; but for the A and C particles it can result from the small class number (k), i.e., k = 8 and k = 13, respectively. However, for larger j values, the $f_3(j)$ and $f_3(D)$ function values are fairly close each other. For the D and E particles, the higher $f_3(j)$ value scatter may result from small particle number (N). For set C the difference between $f_3(j)$ and $f_3(D)$ is the largest one; it is similar to the behaviour of $f_2(d)$ functions (Fig. 3c) and the Dr moments (Table 2c), as well.

TABLE 2E

Moments $\langle D^r \rangle$ (r = 1, 2, 3) for graphite (E), determined by different methods

r	$\langle D^r \rangle$	method I	method II
1	$\langle D \rangle$	$1.44 \cdot 10^{-2}$, mm	$1.35 \cdot 10^{-2}$, mm
2	$\langle D^2 \rangle$	$2.29 \cdot 10^{-4}$, mm^2	$2.15 \cdot 10^{-4}$, mm^2
3	$\langle D^3 \rangle$	$4.10 \cdot 10^{-6}$, mm^3	$3.85 \cdot 10^{-6}$, mm^3

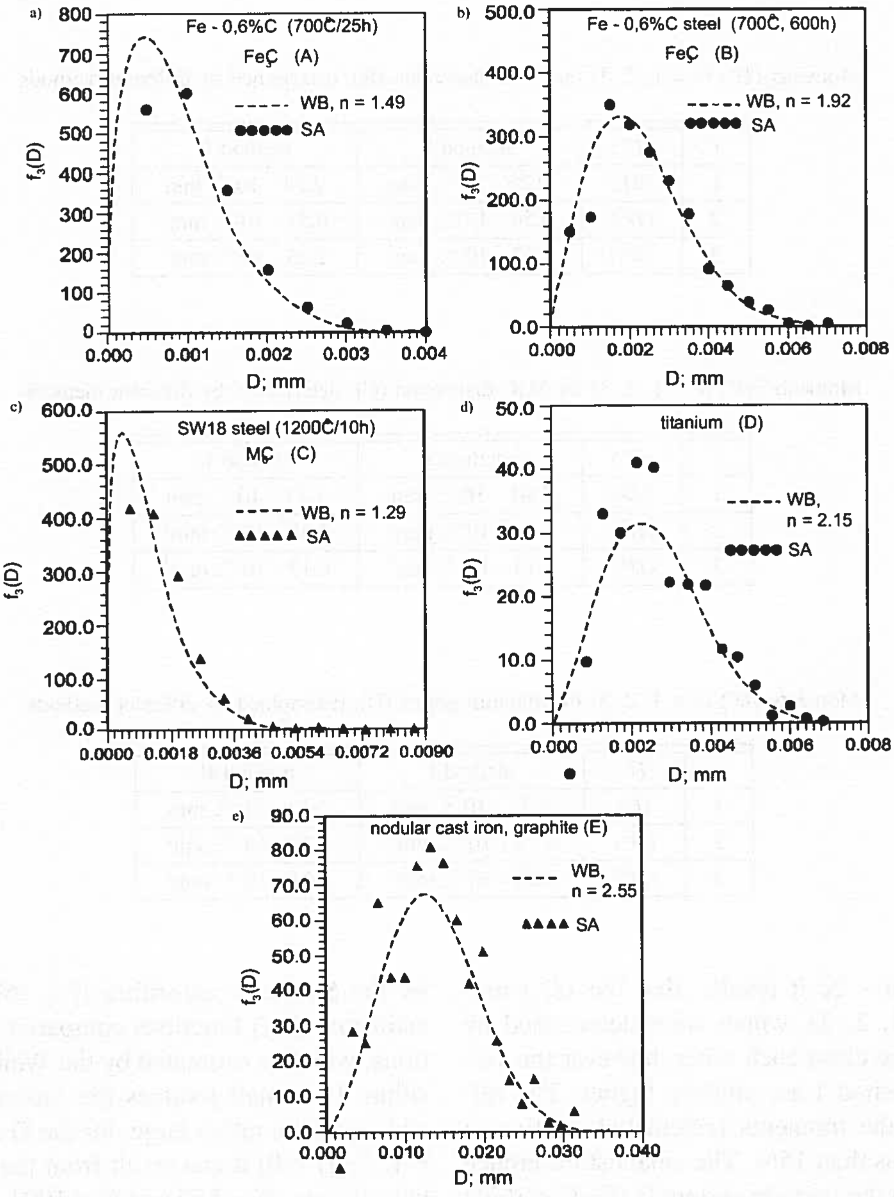

Fig. 4. Comparison of $f_3(D)$ functions, estimated by Weibull stereology and Saltykow (SA) algorithm for steels (a, b, c), titanium (d) and cast iron (e)

In general, the estimated $f_3(D)$ functions belong to the scatter region for the estimated $f_3(j)$ functions; it means, the $f_3(D)$ functions smooth the $f_3(j)$ functions. In conclusion, the results obtained by different algorithms are consistent; however the scatter of the $f_3(j)$ function values is high.

5. Discussion and conclusions

A random set of spheres is described by the D diameter PDF $f_3(D)$ and the $\langle D^r \rangle$ moments. The random set of profiles is described by the d diameter PDF $f_2(d)$ and the $\langle d^r \rangle$ moments. Eqs. (3), (5) and (6) form a mathematical basis for stereological measurement methods in quantitative metallography of spherical particles. The $f_2(d)$ is accessible for direct measurements (in microscope field of view or on microstructure photography).

Given, empirical $f_2(d)$ and the $\langle d^r \rangle$ moments, allow for the determination of: (i) $\langle D^r \rangle$ moments (Eq. (3)); and (ii) $f_3(j)$ function (Saltykow algorithm, Eq. (6)). If the sphere diameters distribution corresponds to the Weibull one, then Eqs. (10), (13) and (14) form the basis for stereological estimation method of the Weibull PDF $f_3(D)$ function parameters (n, α) (the Weibull stereology algorithm).

A particle size distribution corresponds to the Weibull one, if it is possible to approximate the empirical data by a $f_2(d)$ approximating function, which is calculated by the Wicksell equation Eq. (5) (usually in a discrete form of the Satykow's equations – Eq. (6)) including the Weibull PDF $f_3(D)$. Because for the analysed particle systems such an approximation is possible, one can assume, the particle diameters distribution corresponds to the Weibull distribution. The $\langle D^r \rangle$ moments analysis shows that the estimation precision depends on the n parameter of the Weibull PDF $f_3(D)$. In particular, for small n values (n = 1.29 for particle system C) the precision is worse. It is important to notice, that for all particle systems (A-E) the $\langle D^r \rangle$ moments determined by Eq. (3) are larger than those which were determined by the Weibull stereology methods (Table 2a – 2e); it seems, the systematic difference could be connected with the particle form which less or more deviates from sphere, Fig. 1.

Finally, if the n parameter of the Weibull PDF and the k class number in the Saltykow algorithm are not too small (particle systems B, D, E), the $f_3(D)$ functions – estimated with Weibull stereology algorithm – can be used for smoothing of the $f_3(j)$ functions – estimated with Saltykow algorithm.

6. Conclusions

1. If a sphere diameter distribution corresponds to the Weibull distribution, then estimation of the Weibull PDF $f_3(D)$ may be reduced to estimation of its parameters (α, n) by the Weibull stereology algorithm.
2. An empirical particle size (sphere diameter) distribution corresponds to the Weibull one, if the approximating function $f_2(d)$ (given by Wicksell equation Eq. (5) including the Weibull PDF $f_3(D)$) fits to the data (i.e., the empirical $f_2(d)$).
3. As the first approximation, the analysed empirical particle size distributions may be described by the Weibull distribution.

Acknowledgements

The author wish to thank Professor J. Bodziony for discussion and many valuable suggestions.

The financial support from the Polish Ministry of Science High Education, Contract No. 11.11.110.405, is gratefully acknowledged.

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Received: 12 February 2008.