A R C H I V E S O F M E T A L L U R G Y A N D M A T E R I A L S
Volume 55 2010 Issue 2

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THE EFFECT OF THE ERROR OF THERMAL CONDUCTIVITY, SPECIFIC HEAT AND DENSITY DETERMINATION ON THE INVERSE CALCULATION OF THE HEAT TRANSFER COEFFICIENT

OCENA WPŁYWU BŁĘDU OZNACZENIA PRZEWODNOŚCI CIEPLNEJ, CIEPŁA WŁAŚCIWEGO ORAZ GĘSTOŚCI NA WYNIK OBLICZEŃ ODWROTNYCH WSPÓŁCZYNNIKA PRZEJMOWANIA CIEPŁA

The analysis of the effect of the error of thermal conductivity, specific heat and density determination on the inverse calculation of the heat transfer coefficient while water cooling is presented in the paper. The analysis has been based on 2^k fractional design method. The heat transfer coefficient has been calculated from the solution of the inverse heat conduction problem.

W artykule przedstawiono wpływ błędu oznaczenia przewodności cieplnej, ciepła właściwego oraz gęstości na wynik obliczeń odwrotnych współczynnika przejmowania ciepła podczas chłodzenia. Analizy wpływu błędu oznaczenia wybranych parametrów na współczynnik przejmowania ciepła dokonano w oparciu o analizę czynnikową typu 2^k. Współczynnik przejmowania ciepła został obliczony poprzez rozwiązanie równania odwrotnego przewodzenia ciepła przy użyciu programu komputerowego.

1. Introduction

The problem of determination of the boundary conditions is a very important issue for many processes also the metallurgical ones. For example the determination of the heat transfer coefficient is a significant factor in modeling of spray cooling process. This parameter affects the cooling rate and results on the quality of final products. However in many processes, also in water spray cooling, it is not possible to measure the heat transfer coefficient in the direct way. In such a case, it is easier to measure the temperature in the selected points of the investigated sample and on the basis of these measurements define the heat transfer coefficient. Such a problems are called the inverse heat transfer problems. They are defined as ill - posed in the sense that conditions of existence and uniqueness of the solution are not necessarily satisfied and that the solution may be unstable to perturbation in input data [1,4,7]. It means that even small perturbation in input data may lead to growing amplitude of the result of the calculations. This may cause that the received results can be significantly different from expected values. The effect of the error of thermal conductivity, specific heat and density determination on the results of the inverse calculation of the heat transfer coefficient while cooling is analyzed in the paper. The analyses is based on 2^k fractional design method [9], which is widely used in experiments involving several factors on a response. The heat transfer coefficient has been calculated from the solution of the inverse heat conduction problem. A finite element method has been applied to solve the problem [2,3,7]. The simulated temperature variations inside the specimen have been used in place of the measurements for the heat transfer coefficient calculation.

2. Basis of 2^k fractional design

The simplest variation of 2^k fractional uses only two factors, say *A* and *B*, each run at two levels, which may be called "low" and "high". In that case we consider four tests. The scheme of such a design is showed in table 1.

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TABLE 1
Scheme of 2^2 fractional design for two factors without replication

No.	Label	Factor		Treatment Combination
		A	B	
1	*(1)*	–	–	*A* low, *B* low
2	*a*	+	–	*A* high, *B* low
3	*b*	–	+	*A* low, *B* high
4	*ab*	+	+	*A* high, *B* high

In two – level fractional design the following designation are commonly used [5,8,9]:

✓ The tested factor and the effect of that factor is denoted by a capital Latin latter. Thus "*A*" refers the tested factor and the effect of factor *A*, "*B*" refers the factor *B* and the effect of factor *B*. "*AB*" refers to the *AB* interaction.

✓ The low level of one of the factors is denote by "–", the high level of one of the factors is denote by "+".

✓ The treatment combination are usually represented by lowercase letters. Thus, a represents the treatment combination of *A* at high level and *B* at the low level, *b* represents *A* at low level and *B* at high level, and *ab* represents both factors at the high level. By convention, *(1)* is used to denote both factors at the low level.

This notation will be used in the description of 2^k series. In two-level fractional design, we may define the average effect of factor as the change in response produced by a change in the level of that factor averaged over the levels of the other factors. For example the average effect of factor *A* when the factor *B* is at low level is equal *a – (1)*, the average effect of factor *A* when factor *B* is at high level is equal *ab – b*. Now the main effect of factor *A* may be defined using average effects of factors *A* and *B* as follow

$$A = \frac{1}{2}[a - (1) + ab - b] \tag{1}$$

The main effect of *B* as

$$B = \frac{1}{2}[b - m(1) + ab - a] \tag{2}$$

The interaction effect *AB* as the average difference between the effect *A* at high level of *B* and the effect of *A* at low level of *B*. Thus,

$$AB = \frac{1}{2}[ab - b + a(1) - b] \tag{3}$$

The expressions in brackets are called contrast or total effect of one of the factors, namely:

$$\begin{aligned} ContrastA &= a + ab - (1) - b, \\ ContrastB &= b + ab - (1) - a, \\ ContrastAB &= (1) + ab - a - b. \end{aligned} \tag{4}$$

Using contrasts one may calculate the sum of mean squares divisions from the average value of SS_A,SS_B,SS_{AB}:

$$\begin{aligned} SS_A &= \frac{(ContrastA)^2}{2^2} = \frac{(\mathrm{a + ab - (l) - b})^2}{4} \\ SS_B &= \frac{(ContrastB)^2}{2^2} = \frac{(\mathrm{b + ab - (l) - a})^2}{4} \\ SS_{AB} &= \frac{(ContrastAB)^2}{2^2} = \frac{(\mathrm{(1) + ab - a - b})^2}{4} \end{aligned} \tag{5}$$

In a 2^k factorial design, it is easy to express the results of the experiment in terms of the regression model. For two factors each on two levels the regression model is as follow

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon \tag{6}$$

where
y – response,
β_0, β_1, β_2, β_{12} – sought regression coefficient,
x_1 – coded variable that represents factor *A*,
x_2 – coded variable that represents factor *B*,
x_1x_2 – coded variable that represents interaction between two factors *A* and *B*.
ε – the term which results from the random error.
The variables x_1 and x_2 are coded as follow -1 for low level and +1 for high level.

3. Scope of calculations

The first step of the 2^k fractional design was to determine the quantity of examined parameters and set up the boundary of intervals variations. In the considered case the determination errors of three factors (k =3) on the heat transfer calculations while cooling were analyzed. This factors are: thermal conductivity, specific heat and density.

In these case the results of the experiments were simulated by numerical calculations, so the number of replications n in these experiment equals 1 for each set of investigated variables. Because there were three factors examined, the number of tests was equal $2^3 = 8$. All possible combination of the tests are shown in table 2. To clarify the description of tests, in place of the formal denotation of investigated parameters symbols common in heat transfer were used. The thermal conductivity λ represents *A*, specific heat c_p – *B*, and density ρ – *C*.

TABLE 2

The matrix of the experiment

Test number	λ	c_p	ρ	Label
	A	B	C	
1	–	–	–	*(1)*
2	+	–	–	*a*
3	–	+	–	*b*
4	+	+	–	*ab*
5	–	–	+	*c*
6	+	–	+	*ac*
7	–	+	+	*bc*
8	+	+	+	*abc*

Three kinds of materials were analyzed. Copper, brass and inconel (the alloy of nickel, chromium and iron) have been chosen. The thermal conductivity of these materials differs significantly but the specific heat is very similar. The numerical calculations have been performed in order to simulate the temperature variation in cylindrical samples subjected to cooling. The following assumptions have been made:

- specimen length L = 150 mm,
- only the face surface of sample is cooled. The side surface and the rear end of sample are perfectly insulated,
- constant heat transfer coefficient α = 10 kW/(m^2·K) is specified on the cooled surface,
- the heat is conducted in the direction of the specimen axis only,
- the temperature distribution is simulated at the point placed 2 mm below the cooled surface.
- the initial sample temperature is t (x,0) = 700°C
- the cooling medium temperature is t_{env} = 20°C.

The computed data have been used in place of the measurement. The computation have been performed to compare the influence of the determination error of thermal conductivity, specific heat and density on the heat transfer coefficient value which is obtained from the inverse solution to the heat transfer problem.

It is assumed that thermophysical properties of the sample materials do not depend on temperature. In table 3 the thermophysical properties of the investigated materials have been presented [6].

TABLE 3

Thermophysical properties of the materials investigated by the fractional design method

Material	Thermal conductivity	Specific heat	Density
	λ, W/(m·K)	c_p, kJ/(kg·K)	ρ, kg/m^3
Copper	401	380	8920
Brass	101	377	8600
Inconel	14,9	444	8470

4. Course of calculation

The initial step in the inverse calculation of the heat transfer coefficient is the choice of the approximation function. The function should allow to interpolate the heat transfer coefficient in the time interval from 0 to 40 s. First, the heat transfer coefficient was approximated in the whole time interval by the second order polynomial:

$$\alpha(\tau) = M_1 + M_2\tau + M_3\tau^2 \tag{7}$$

and by the linear one:

$$\alpha(\tau) = M_1 + M_2\tau. \tag{8}$$

The results of calculations for these tests have been presented in figures 1a and 1b.

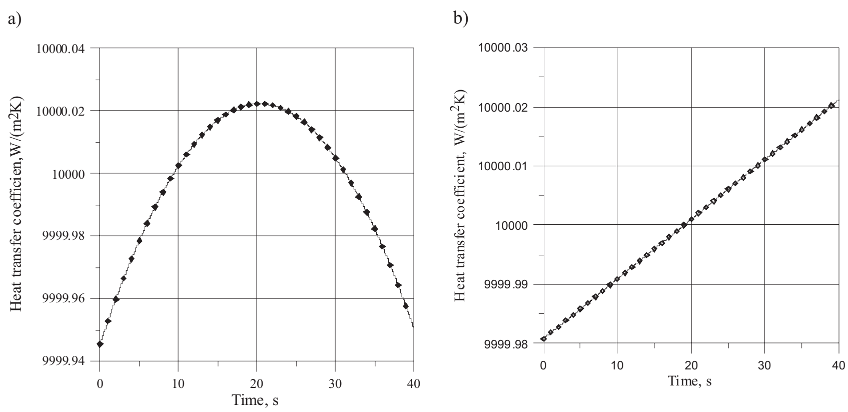

Fig. 1. Variation of the heat transfer coefficient approximated by the first and second order polynomials

The calculation performed let to find out, that the parameters M_2 and M_3 have very small values equals almost to zero, both for the first and second order polynomials. It clearly indicates that the sought approximating function is constant. In the second set of calculations the heat transfer coefficient has been approximated by the constant M_1 over the whole time interval. The value of $M_1 = \alpha = 9999.9956$ W/(m^2·K) has been obtained based on inverse analyses.

The relative error of the heat transfer coefficient is 0.00004%. These results let us to conclude, that sought heat transfer coefficient has a constant value.

For fractal design it was assumed that the error of thermal conductivity and specific heat determination for all investigated materials oscillate between 12 and 13 %. The 6% error of density determination has been assumed. The values of thermophysical parameters used in the fractional design are shown in table 4.

TABLE 4

Thermophysical properties of the materials investigated by the fractional design method

Material	Thermal conductivity		Specific heat		Density	
	λ, W/(m·K)		c_p, kJ/(kg·K)		ρ, kg/m^3	
	Low level	High level	Low level	High level	Low level	High level
Copper	351	451	330	430	8420	9420
Brass	88	114	327	427	8100	9100
Inconel	13	16,9	394	494	7970	8970

Now, the temperature variations 2 mm below the sample surface have been simulated following the tests specified in table 2 for the material properties shown in table 4. Next, the simulated temperature distributions have been used in the inverse calculation of the heat transfer coefficient for each test.

5. Results

The errors of the inverse calculations of the heat transfer coefficient are shown in Figure 2. The results have shown, that the greatest relative error of the heat transfer coefficient for brass cooling is observed if all three factors are at the same level. These tests have been indicated in Table 2 as *(1)* and *abc*. The same relation is observed for copper, Figure 2. If the material properties as specified in Table 2 as test *(1)* and *abc* are used in the inverse calculation, it will result in the error of the heat transfer coefficient calculation at the level of 17%. For the tests indicated as *a*, *b* and *c*, where only one parameter was at the high level, the relative error of the heat transfer coefficient calculation is lower. In the case of the test *(a)* the error for copper is 5% and 7% for brass. For the test *(b)* the relative error is 2% for copper and 0.6% for brass. In the case of the test *(c)* the errors for copper and brass are at the level of 10%, Figure 2. In case of inconel for the test *(bc)* the relative error has the greatest value of 31.27%. High values of the relative error are observed also for the tests indicated as *(1)* and *a*.

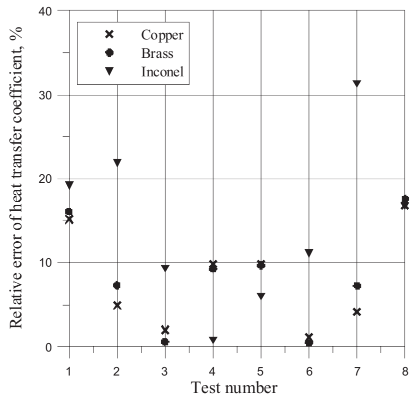

Fig. 2. The relative errors of the inverse calculation of the heat transfer coefficient for the test selected for the fractional design method

In Figure 3 the results the variance analysis are presented. It has been observed for all the tests that the greatest source of variation of the heat transfer coefficient calculation is the specific heat indicated as the factor *B*. The maximum participation of the error of the specific heat determination in the total error of the heat transfer coefficient calculation is 55% in case of cooper and 70% in case of brass and inconel (Figure 3).

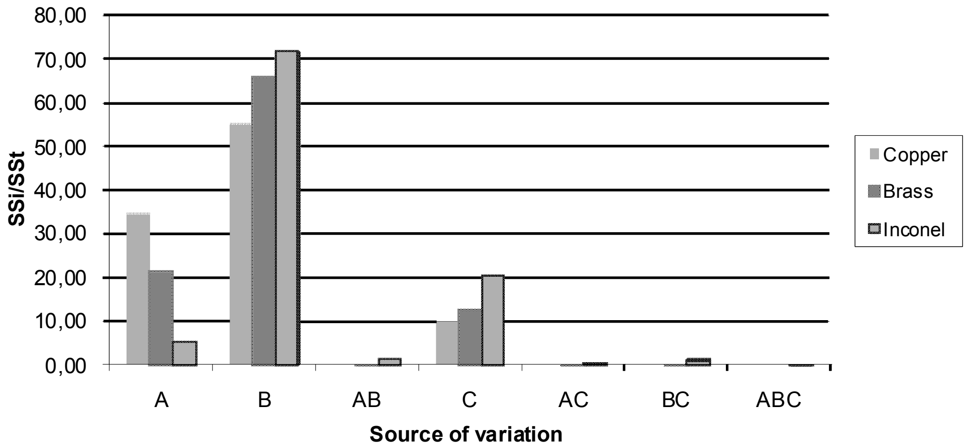

Fig. 3. The participation of the determination error SS_i of the thermal conductivity, specific heat and density in the total error of inverse calculation SS_T of the heat transfer coefficient for all sources of variation

The participation of the inaccuracy of the thermal conductivity determination in the total error of the heat transfer coefficient calculation is 35% for copper, 21% for brass and 5% for inconel. It can be noted, that if the thermal conductivity is higher, the greatest is its participation in the total error of heat transfer coefficient calculation.

Incorrectly determined density results in the total error of the heat transfer coefficient calculation at the level of 10 – 12% in case of copper and brass, and at about 20% for inconel.

The effect of interaction of all the investigated factors was very small, for the investigated materials. It is less than 1% (Figure 3). Only in the case of inconel the interaction between thermal conductivity and specific heat (the interaction *AB*) and between specific heat and density (the interaction *BC*) is noticeable but very small (Figure 3).

The regression equation (6) has been used to analyze series of tests. For example the regression equation for brass has the following form

$$y = 9971 + \left(\frac{949,95}{2}\right)x_1 + \left(\frac{1672,94}{2}\right)x_2 + \left(\frac{732,39}{2}\right)x_3 + \left(\frac{53,07}{2}\right)x_1x_2 + \left(\frac{23,08}{2}\right)x_1x_3 + \left(\frac{73,40}{2}\right)x_2x_3 + \left(\frac{0,44}{2}\right)x_1x_2x_3. \tag{9}$$

This equation let us to calculate the value of searched parameter for different values of the input data. It should be noted, that these equation does not has the element of statistical error of physical experiment, which would be present while cooling of the investigated samples. It is due to numerical simulation of the measurements.

The regression equation allows to calculate the value of the searched parameter, if coded values (- 1 or +1) are used in place of variables x_1, x_2, x_3. For example for the test *(1)*, Table 2, it can be readily calculate:

$$y = 10218 + \left(\frac{949,95}{2}\right)(-1) + \left(\frac{1672,94}{2}\right)(-1) + \left(\frac{732,39}{2}\right)(-1) + \left(\frac{53,07}{2}\right)(-1)(-1) + \left(\frac{23,08}{2}\right)(-1)(-1) + \left(\frac{73,40}{2}\right)(-1)(-1) + \left(\frac{0,44}{2}\right)(-1)(-1)(-1) = 8367. \tag{10}$$

The calculated value of the heat transfer coefficient is with a good agreement with the numerical test result, which has given α=8368 W/(m^2·K).

6. Summary

Application of the fractional design method let us to determine the effect of the error of the thermo physical properties determination on the computed value of the heat transfer coefficient. It was found out, that for the assumed data the greatest influence on the final value of the heat transfer coefficient has the inaccuracy of the specific heat determination. The determination error of specific heat has 0.55 to 0.72 participation in the total error of heat transfer coefficient computation. The determination error of the thermal conductivity has 0.05 to 0.35 participation in total error and incorrectly determined density causes 0.1 to 0.2 participation at the total error of the heat transfer computation.

It can be conclude, that for the investigation of the heat transfer coefficient it is very important to determine precisely the thermo physical properties, especially the specific heat of the investigated material.

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Received: 10 December 2010.