P. MACIOŁ*, J. GAWĄD*, D. PODORSKA*

ARRANGEMENT OF FLOW MODIFICATION DEVICES IN CONTINUOUS CASTING TUNDISH BASED ON MULTICRITERION OPTIMIZATION

DOBÓR UKŁADU PRZEGRÓD W KADZI POŚREDNIEJ CIĄGŁEGO ODLEWANIA STALI ZA POMOCĄ OPTYMALIZACJI WIELOKRYTERIALNEJ

The article presents the method of optimization of flow modifying devices arrangement in the casting tundish, which create the favourable conditions for removal of inclusions. The numerical simulation of velocity field of steel flow in the tundish is coupled with the optimization procedure, consisting in search for Pareto-optimal set of geometric parameters of the tundish. The problem under study concerns the system of two lower and one upper dams. The calculations employing finite elements method were carried out in the two dimensional system. The simulation model was constructed on the basis of ADINA-F commercial program. Optimization criteria were formulated for the residence time of steel in the tundish and the velocity components in the sub-surface layer. The codes developed by the authors were applied, which allowed for fully automatic coupling of optimization module with ADINA-F program. As a result the set of six geometric parameters describing the optimal dams arrangement was obtained.

Keywords: steel casting, casting tundish, non-metallic inclusions, velocity field, mathematical model

Artykuł przedstawia metodę wyznaczania optymalnego rozmieszczenia przegród kształtujących przepływ w kadzi pośredniej odlewania ciągłego pod względem korzystnych warunków dla usuwania wtrąceń niemetalicznych. Numeryczna symulacja pola prędkości przepływu stali w kadzi pośredniej została sprzężona z procedurą optymalizacyjną, polegającą na znalezieniu optymalnego w sensie Pareto zbioru parametrów geometrycznych kadzi. Obiektem optymalizacji był układ dwóch przegród dolnych i jednej górnej. Obliczenia przy pomocy metody elementów skończonych zrealizowano w układzie dwuwymiarowym. Model symulacji został zbudowany na bazie komercyjnego programu ADINA-F. Kryteria optymalizacji zostały wyrażone poprzez czas przebywania stali w kadzi oraz przez składowe prędkości stali w warstwie powierzchniowej. Użyto kodów opracowanych przez autorów, co pozwoliło na całkowicie automatyczne sprzężenie modułu optymalizującego z programem ADINA-F. W wyniku obliczeń uzyskano zestaw sześciu parametrów geometrycznych, opisujących optymalny układ przegród.

1. Introduction

Production of good quality steel, which fulfils the requirements resulting from its applications as the final product, needs the effective control of the properties of liquid steel supplied to continuous casting machine as well as the parameters of casting. The main factors determining the quality of steel ingot are: the casting temperature, content of non-metallic inclusions, the position and stability of liquid steel meniscus and composition and properties of casting powder [1]. First three of these parameters are usually controlled in a casting tundish, which is becoming important metallurgical reactor, in which the steel is finally treated prior to casting in a mould.

The operation of casting tundish is characterized with the flow pattern and resulting residence time of liquid steel, which in turn control the removal of non-metallic inclusions. The inclusions, mainly oxides are the unavoidable result of steel deoxidisation in industrial conditions are still present in considerable amount in the casting ladle. An additional amount of inclusions is being formed during lowering the temperature of steel as a result of shift of thermodynamic equilibrium between steel and oxide phase. The technological experience shows that if the ladle operation is effective in inclusions removal, it is also effective in flow tranquillising and temperature equalizing. The inclusions are removed according to two modes: swimming out to the top slag or

* FACULTY OF METALS ENGINEERING AND INDUSTRIAL COMPUTER SCIENCE, AGH UNIVERSITY OF SCIENCE AND TECHNOLOGY, 30-059 KRAKÓW, 30 MICKIEWICZA AVE., POLAND

sticking to the tundish walls [2, 3]. Thus, the flow pattern in the tundish should promote these two processes. The zone of strongly turbulent flow, in which the inclusion particles coagulate, should be restricted to the tundish region close to the ladle outflow. In the prevailing part the tundish volume the plug flow is expected, which allows for the swimming out the inclusions and their absorption in top slag. The flow pattern of liquid steel in the tundish in many cases is established by means of so called flow modifying devices (FMD). The optimum dimensions and positions of these devices are the question, which can be answered on the ground of physical modelling, industrial experiments and numerical simulations. The numerical methods allow for determination of steel velocity field in the tundish of chosen geometry. The approximate solutions based on the theory of elementary reactors [4, 5], as well as complete finite element solutions of Navier – Stokes equation [6, 7], were obtained for several cases. The aim of the present work is the demonstration of the possibilities arising from the application of optimisation procedure for designing arrangement of the FMD in two strand tundish. Two kinds of vertical devices were taken into consideration: upper dam and lower dam.

2. Numerical model

The numerical modelling of physicochemical processes in metallurgy of steel is an exceptionally complex task. At the present level of development of numerical methods, full numerical analysis of processes occurring in majority of metallurgical reactors is very difficult. For this reason, several simplifying assumptions were made in FEM solution applied in the present work. It is obvious that the analysis involving more phenomena results in higher computation costs. As the procedure of optimization requires multiple solutions of complex numerical problem, the simplifications of numerical model are necessary. On the basis of significance analysis of respective process parameters the following assumptions were accepted for the purpose of the present work:

• The modelled system is regarded as two dimensional, symmetrical with respect to shorter axis of the tundish. This assumption leads to decrease of agreement between numerical solution and real process, resulting mainly from the lack of symmetry in positions of outflow ports with regard to longer axis of the tundish, and from variation of metal flow with the distance from the tundish walls. However, the three dimensional solution would require enormously long calculation time. Calculation time is a great obstacle especially in optimization cases, as FEM simulations are carried out for each optimisation step, so cumulated computation time could be extremely long.

• The model of incompressible viscous liquid of constant viscosity is applied. The viscosity of 1 kg/m·s and density equal to 7000 kg/m^3 were assumed. This simplified description of the material was considered adequate, because presented FEM solution does not solve real industrial problem, and qualitative agreement is sufficient for optimization procedure. The assumption of liquid metal incompressibility seems fully justified. The measurable effects of liquid steel compressibility do not appear at the velocities significantly lower than sound velocity. Assumption of constant viscosity allows to neglect the effects resulting from the variation of temperature and chemical composition of steel. The influence of temperature field in the tundish on viscosity and indirectly on the whole process is of some importance. However, including the temperature field in the analysis would considerably increase the complexity of the problem and time of calculations. For this reason constant temperature of steel in the tundish was assumed in the present case. Moreover, the dependence of viscosity on chemical composition of steel and on amount of non-metallic inclusions cannot be introduced into calculations, what is due to the lack of appropriate models as well as experimental data.

• The model does not take into account the physical interaction between metal and slag at their interface. The behaviour of slag is also neglected at the level of FEM calculations. However, the criteria of optimization involve the properties of metal surface layer, which are correct from the point of view of metallurgical process.

• The details of impact pad in the inlet region of tundish are not taken into account in FEM calculations.

• The considered model of casting tundish does not take into account the zone of strong turbulent mixing in the inflow region. This zone is as a rule partly isolated from the main volume of tundish by means of turbulence inhibitor ("turbostop") or lower dam [8], or alternatively the upper dam [9]. Turbulent mixing zone plays important role in removal of inclusions. The turbulent collision of inclusions particles is the main mode of their coagulation. Larger particles are more susceptible to swimming out from steel and assimilation in top slag.

• The simulated process is transient. The steel flow through casting tundish is stationary during the majority of process duration, thus the assumption of stationary process might be justified. However, one of the optimization criteria adopted in the present work requires the residence time of elementary metal volume in the tundish to be maximal. For this reason the transient solution was applied.

• No-slip boundary condition is assumed on tundish wall, as well as on the surfaces of FMDs. Metal surface is treated as a wall with full slip boundary condition.

The numerical model of process was built on the basis of commercial ADINA-F program, which employs the finite elements method. The geometry of tundish chosen in the present work is similar to several constructions appearing in the industrial conditions, e.g. Mittal Steel Poland, Kraków Branch. The considered tundish corresponds to two-strand slab caster with two surfaces of symmetry. Liquid steel is poured through central shrouded inlet. In the present work the complete solution of 2D Navier-Stokes equation was carried out:

$$\begin{cases} \dfrac{\partial p_{yy}}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z} + Y\rho = \rho \dfrac{\partial v^y}{\partial t} \\ \dfrac{\partial \tau_{zy}}{\partial y} + \dfrac{\partial p_{zz}}{\partial z} + Z\rho = \rho \dfrac{\partial v^z}{\partial t} \end{cases} \tag{1}$$

where $p_{\text{ii}}, \tau_{\text{ii}}$ – stresses, ρ – density, Y, Z – gravitation forces, v^y, v^z – velocities in y, z directions, respectively.

The three- and four-node finite elements were used. The simulated steel flow in the tundish was modified with three FMD in the form of vertical dams. Their positions in the tundish were assumed as variables and subject to optimization. The dimensions of the tundish including arrangement of the dams are shown in Fig. 1. The typical mesh of finite elements is presented in Fig. 2. An additional layer of elements is introduced at the interface of liquid metal and slag. During the optimization procedure this layer is treated as a region, where removal of inclusions takes place. Since vertical velocities at the interface between liquid metal and slag are equal to zero (due to slip wall boundary condition), the values of velocity taken into account during optimization are attributed to the nodes laying below this additional layer (Fig. 1). Finite element mesh, generated on this layer have the thickness of one element (10 mm) and variable number of elements along the length.

The residence time of steel in the tundish is determined by means of the tracer procedure. Mass transfer model coupled with Navier-Stokes solution is utilized for determination of tracer distribution. Tracer is introduced as virtual material. This additional material has no influence on velocity field of metal. The virtual tracer is introduced to the system at the moment, when the system reaches stationary state and then is distributed in the system due to convection. However, small artificial diffusion is introduced due to numerical convergence reason. Total amount of 0.01 units of the tracer are added during three time steps between 30 an 33 second of the process duration. The unit of the tracer is a mass ratio of tracer to the total mass of the fluid in an elementary volume:

$$\text{mass ratio}_{\text{tracer}} + \text{mass ratio}_{\text{steel}} = 1.$$

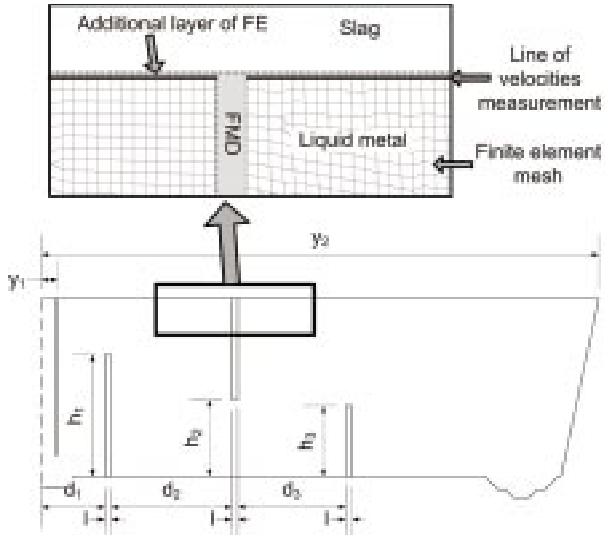

Fig. 1. Geometric dimensions of the tundish. Only the quarter is presented

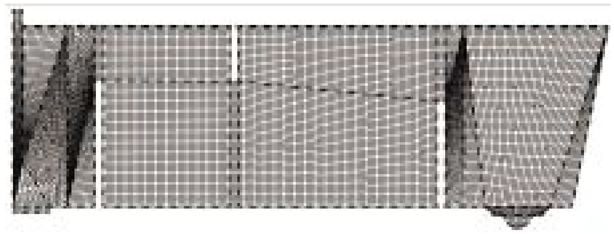

Fig. 2. Example of FEM mesh for the model of casting tundish

Quantity of virtual tracer is recorded in additional layer. Tracer is used only for measuring of material points' paths in flowing metal.

ADINA-F program allows the operation in batch mode, receiving the input data and recording the results in text files. This makes the coupling of FEM model with the optimizing program possible. The input data are prepared in a parametric form, which allows for fully automatic generation of FEM models for various configurations of dam arrangement, as well as automatic transmission of calculations results to optimizing programme. In the present project, the calculations were performed in the cluster environment, consisting of 9 computational nodes.

3. Optimization

The problem of finding the optimal positions of dams in the tundish is formulated as a task of multi-criterion optimisation [10]:

$$\min \mathbf{F}(\mathbf{X}) = \{\phi_1(\mathbf{X}), ..., \phi_i(\mathbf{X}), ..., \phi_m(\mathbf{X})\} \tag{2}$$

where $\mathbf{X} = \{x_1 ... x_n\}$ – the vector of decision (design) variables in the space of requested solutions, $\phi_{i,}$, $i = 1...m$ – the component of $\mathbf{F(X)}$ vector belonging to criteria space and representing single criterion.

The multicriterion formulation is suitable for the problems consisting in conflicting and possibly noncommensurable (i.e. having different units) criteria, which should be simultaneously minimized. As it is shown in further part of this section, the considered problem falls into such group of issues.

For the problem under study the vector of decision variables contains 6 parameters, which describe the position and height of dams in casting tundish:

$$\mathbf{X} = \{d_1, h_1, d_2, h_2, d_3, h_3\} \tag{3}$$

where: d_1, d_2, d_3 – the distances between dams and h_1, h_2, h_3 – the height of the dams (Fig. 1).

In order to put the problem as a task of multicriterion optimization the potential conflicts between criteria had to be taken into account, i.e. impossibility of obtaining simultaneously the minimum values for all components of vector $\mathbf{F}$. The solution of minimization task is then the set of compromise solutions, termed Pareto set. For this purpose the definition of vector $\mathbf{U}$ domination over vector $\mathbf{V}$ (denoted as $\mathbf{U} \prec \mathbf{V}$) in the Pareto meaning is introduced:

$$\begin{aligned} &\mathbf{U} \prec \mathbf{V} \Leftrightarrow \forall i \in \{1, \ldots, m\}, \\ &u_i \leqslant v_i \wedge \exists j \in \{1, \ldots, m\} : u_j < v_j \end{aligned} \tag{4}$$

The solution $\mathbf{U}$ is termed Pareto-optimal if there is no solution $\mathbf{V}$ for which $\mathbf{F}(\mathbf{V}) \prec \mathbf{F}(\mathbf{U})$. Thus, the solution of optimisation task (4) consists in construction of the set of Pareto-optimal solutions. The set of Pareto optima represents those feasible solutions where no criterion can be improved without deteriorating at least one other criterion. In the case of non-conflicting criteria, the Pareto set includes single solution. Otherwise, the set consists of the compromise solutions by considering trade-offs between the competing criteria.

In the present work three criteria of solution evaluation were defined. First of them is used as a measure of residence time of steel in the tundish:

$$\phi_1 = \int_{t_1}^{t_2} \int_{y_1}^{y_2} C dy dt \tag{5}$$

where: C – concentration of tracer in sub-surface layer determined with the coordinates y_1 and y_2, t_1 and t_2 – the time of start of recording tracer concentration and the time of termination, respectively.

Criterion ϕ_1 expresses how long the tracer is held in the vicinity of interface between liquid metal and the slag. Despite no direct physical meaning could be related to that criterion, the maximization of ϕ_1 is required. The technological knowledge suggests that elongated presence of the material in this zone increases the chance of rising the inclusions to the surface.

Remaining two criteria of solution valuation are based on the values of components of liquid metal flow velocity in the sub-surface layer, mentioned in numerical model description. From the metallurgical point of view, the most desirable flow pattern in the sub-surface layer promotes removal of the inclusions. The low velocity results in lower coagulation rate, leading in turn to the reduced purification of the metal. On the other hand, too high velocity of the metal is also unfavourable, due to the risk of disruption of the slag cover, as well as snatching particles of slag into the liquid metal. In the present paper the assumption was made that components v^y and v^z would fit the acceptable limits. Consequently, the corresponding criteria represent deviation of the calculated nodal velocities from the acceptable values:

$$\phi_2 = \left(\frac{1}{n} \sum_{i=1}^{n} f\left(v_i^y\right) \right)^p + 1 \tag{6}$$

$$\phi_3 = \left(\frac{1}{n} \sum_{i=1}^{n} f\left(v_i^z\right) \right)^p + 1 \tag{7}$$

$$f(v) = \begin{cases} (a - v) : & v < a \\ \quad 0 \quad : & v \in (a, b) \\ (v - b) : & v > b \end{cases} \tag{8}$$

where: n – amount of grid nodes in sub-surface layer, i – index of the FE node, v^y, v^z – horizontal and vertical components of nodal velocity vectors, respectively, a, b – upper and lower limits of tolerable values of velocity components, p – coefficient that allows overlinear impact of the criteria ϕ_2 and ϕ_3. The values of a and b coefficients are chosen basing on industrial and laboratory practice. The value of coefficient p was selected on the basis of numerical experiments in order to achieve faster convergence of optimization method.

For the problem considered in the present work the optimisation task is formulated as the minimization of vector $\mathbf{\Phi}$ of three components:

$$\mathbf{\Phi}(\mathbf{X}) = \{(-\phi_1 + P_1), (\phi_2 + P_2), (\phi_3 + P_3)\} \tag{9}$$

where: P_i – penalty function for i^{th} criterion, taking the non-zero value in the following cases:

- the loss of convergence in FEM solution,
- the vector $\mathbf{X}$ describes the FMD arrangement, which is impossible for geometrical reasons,
- the distance between dams is less than $d_{min} = 0.05$.

The last constraint was introduced because of limitations in FEM grid generator.

The iterative strategy was employed in order to approach the Pareto set. The scalar target function Ψ is defined, basing on the components of vector $\mathbf{\Phi}$. The simplex method was chosen as appropriate for minimization of scalar function Ψ. The Pareto-optimal solutions found in the course of search for Ψ function minima are included into the set of solutions. The authors used two equations, which were employed in construction of scalar target function:

$$\Psi_1 = \frac{\sum_{i=1}^{n} w_i \phi_i(\mathbf{X})}{\sum_{i=1}^{n} w_i} \tag{10}$$

$$\Psi_2 = \max\left(\frac{\phi_i(\mathbf{X}) - \phi_i^{min}}{\phi_i^{min}}\right) \tag{11}$$

where: w_i – the weights attributed to individual criteria ϕ_i; ϕ_i^{min} – the component of criteria for hypothetical ideal solution $\mathbf{X}^*$.

The function (10) is widely applied relation, making possible the calculation of the target function as the weighted average from all criteria. Equation (11) expresses so called "minimax strategy", which consists of minimization of maximal distance from the vector of minimal criteria $\mathbf{\Phi}(\mathbf{X}^*)$: $\phi_i^{min} = \phi_i(\mathbf{X}^*)$. All components ϕ_i of vector $\mathbf{\Phi}(\mathbf{X}^*)$ take the lowest possible values. The vector ϕ_i^{min} is most frequently constructed through optimization, which is carried out separately for each criterion ϕ_i. For the sake of calculation time economy, the approximate solution is applied, which estimates the components of vector ϕ_i^{min} in the course of optimization.

Since the simultaneous application of all solutions from Pareto set is usually impossible, in the present work the index of valuation of solution is introduced, which is based on the distance between the given solution and estimated components for ideal solution:

$$J = \sqrt{\sum_{i=1}^{n}\left(\phi_i(\mathbf{X}) - \phi_i^{min}\right)^2} \tag{12}$$

The quality index given by equation (12) is used only at the final stage of optimization. The components ϕ_i^{min}, which were estimated during optimization procedure, are used. In contrast to the goal function (11), the quality index (12) allows selection of the single final solution on the basis of several Pareto sets and the final estimation of ϕ_i^{min}.

4. The results

The calculations were carried out for the series of the sets of initial parameters, regarding the positions and heights of all three dams in the tundish. The procedure of optimization was performed several times, starting form different initial solutions and resulting in usually different Pareto sets. The entire optimization procedure required launching the simulation jointly ca. 2000 times. The usefulness of both ADINA and the own code in the cluster computational environment was fully confirmed. Table 1 contains the values of parameters for criteria (6)–(8). The values of geometric parameters h_1, h_2 and h_3 were restricted to the range (0.01-0.8 m).

TABLE 1

Parameters for equations (6)–(8)

Criteria	a	b	p
ϕ_2	-0.2	0.1	1.5
ϕ_3	-0.02	0.02	1.5

The typical variation of Pareto set cardinality during consecutive steps of simplex method is presented in Fig. 3a. In the course of optimization, multiple non-dominated solutions were determined. The mean values of the geometric parameters and the criteria as well as their standard deviations at selected step of the optimization are shown in Fig. 3b. It should be emphasized in that figure that Pareto set consisted of very dissimilar solutions during the optimization. This is a clear evidence that the conflicting criteria were present in the optimization problem (9).

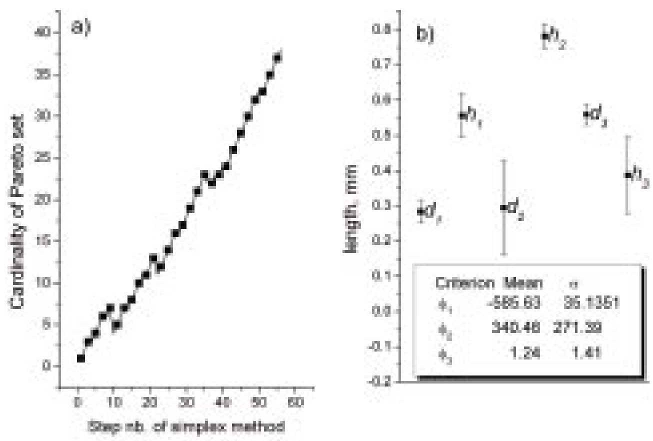

Fig. 3. Illustration of conflicting criteria in the considered optimization problem: a) typical variation of Pareto set cardinality during consecutive steps of simplex optimization method, b) descriptive statistics (mean values and standard deviations σ) calculated for the geometrical parameters and the criteria at 50[th] step of simplex method. The standard deviations of geometrical parameters are denoted by the bars

TABLE 2

The comparison of two the most preferable solutions with respect to the valuation criterion (12): initial solution (row 1) and final solution (row 2)

Lp.	d_1[m]	h_1[m]	d_2[m]	h_2[m]	d_3[m]	h_3[m]	ϕ_1	ϕ_2	ϕ_3
1	0.239	0.7248	0.5618	0.6926	0.5370	0.5136	641.86	866.91	4.87
2	0.406	0.0118	0.0510	0.6598	0.0546	0.0100	1002.13	10.50	1

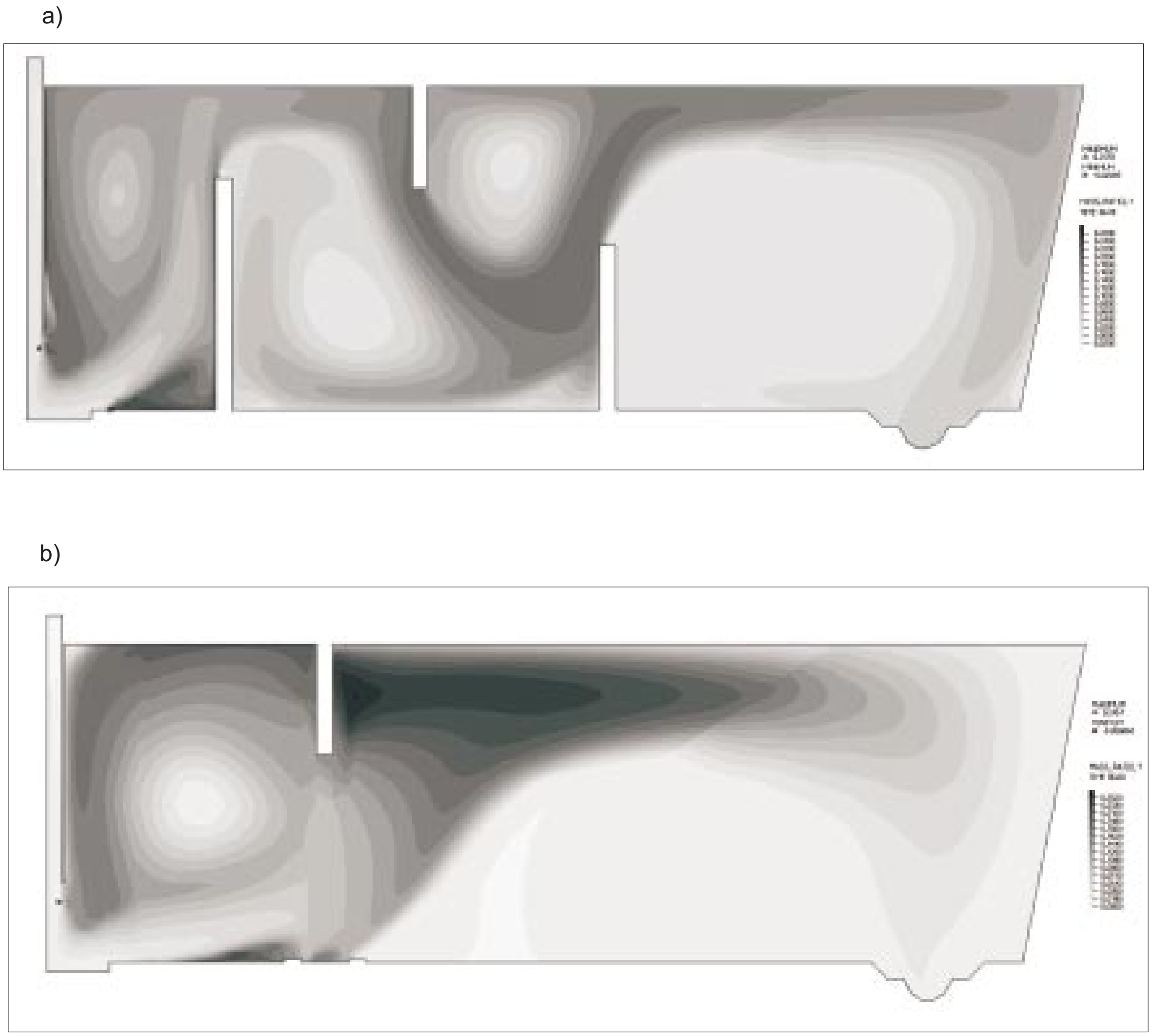

Fig. 4. The arrangement of dams in the tundish according to initial (a) and final solution (b). The distribution of the tracer at t = 36s is presented as the concentration isolines (in mass ratio, defined in description of numerical model)

Nine Pareto-optimal solutions were obtained as a final result of the optimization. Table 2 contains the comparison of the two best solutions with respect to valuation index (12). The best initial solution (Table 2 row 1) was compared to the best final solution (Table 2 row 2). The values of criteria given in Table 2 reveal that the successful minimization of all criteria was obtained. The arrangement of FMD as well as the distribution of the tracer for initial and final solutions is shown in Fig. 4.

Fig. 5a presents the integral of tracer concentration calculated for sub-surface layer, for both initial and final solutions. The single strong peak is observed on the curves. Such an effect is typical for the selected method of the tracer addition to the system [6] and its occurrence is the premise that the FE model reproduces properly the behaviour of the simulated system. It is seen in the Fig. 5 that for the final solution tracer concentration maintains higher value for a longer period. This observation is confirmed by the corresponding values of criterion ϕ_1, increasing in consecutive time steps (Fig. 5b). Such behaviour is favourable for inclusions removal to a top slag.

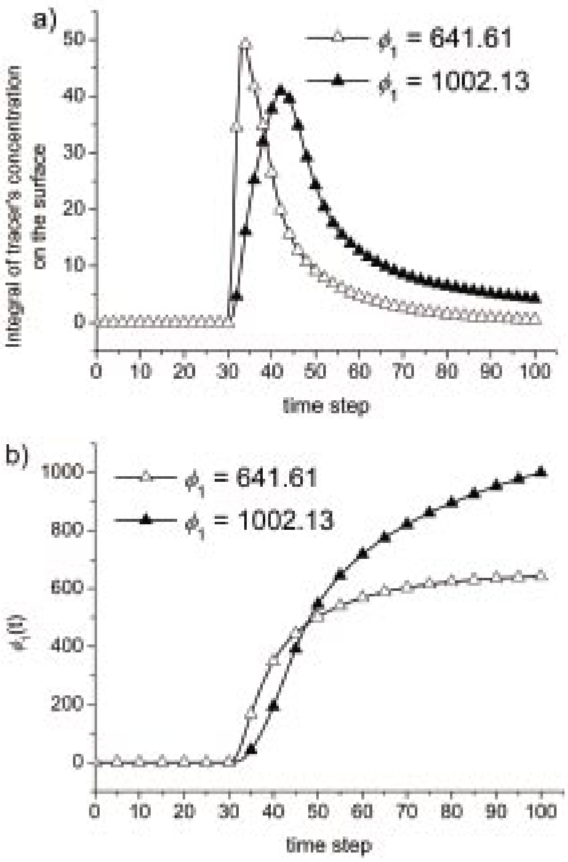

Fig. 5. Integral of tracer concentration (a) and corresponding values of ϕ_1 criterion (b) in subsequent time steps. Open symbols correspond to the initial solution, while filled symbols to the final solution

Fig. 6 demonstrates the comparison of velocity components v^y and v^z for initial and final solutions. The flow pattern for the best initial solution is unacceptable with respect to the parameters specified in Table 1 and marked with horizontal dashed lines in Fig. 6. It is apparent that as a result of optimization the values of velocity components were moved to the acceptable regions. It is also seen in that figure that distribution of velocities is much smoother in the case of final solution. Typical distribution of flow velocity in the tundish is presented in Fig. 7.

The arrangement of dams in all obtained Pareto-optimal solutions is similar. The typical arrangement is presented in Fig. 4b. It is interesting that in this solution the heights of two lower dams h_1 and h_3 are reduced to lower acceptable limit, which implies their elimination. Simultaneously, the highest values of tracer residence time in the tundish were obtained. For the obtained Pareto set the correlation of construction parameters and valuation criteria were carried out. The highest values of correlation were obtained for the parameters d_1 and h_2 (R^2 = 0.961) and criteria ϕ_1 and ϕ_2 (R^2 = 0.956).

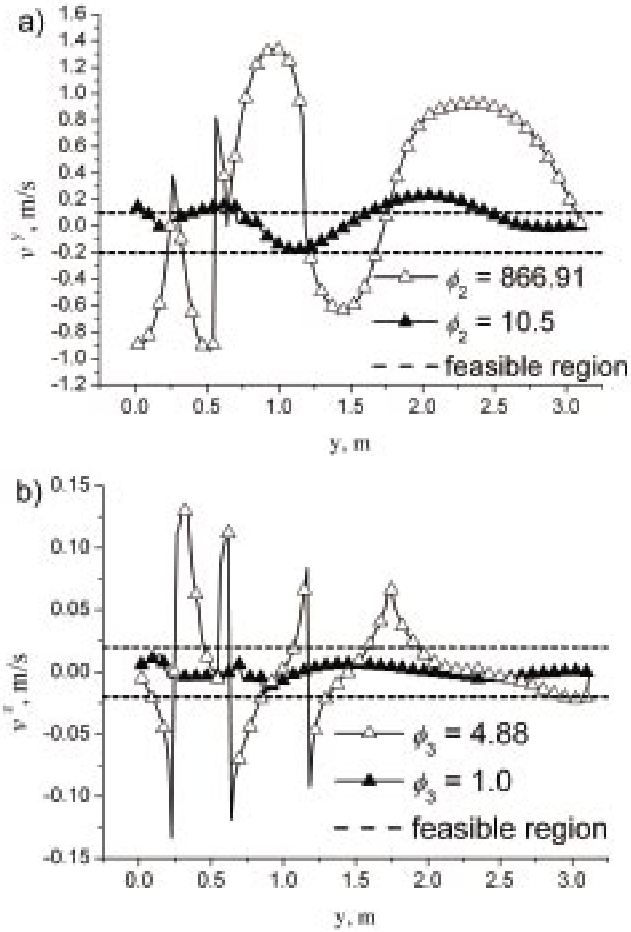

Fig. 6. The velocity distribution in sub-surface layer and corresponding values of ϕ_1 and ϕ_2 criteria: a) v^y component, b) v^z component. Open symbols – initial solution, filled symbols – final solution. The range of acceptable values were marked with dashed line

Fig. 7. Distribution of velocity magnitude at t = 36s for initial (top) and final (bottom) solution

The obtained results suggest that presented optimization approach allows obtaining of non-obvious results. The arrangement that requires only one dam instead of three FMD is remarkably better due to economical reasons.

5. Conclusions

In the present work the method of multicriterion optimization was adopted to solve the problem of optimal arrangement of flow modifying devices in casting tundish. Application of multicriterion optimization seems fully adequate due to noncommensurable and conflicting criteria defined for evaluation of the solutions. The methodology worked out allows for considerably unbounded choice of numerical models of casting tundish as well as the valuation criteria of solutions. The obtained results prove the suitability of the above method in the search for optimum arrangement of dams. The solutions found in the course of calculations are evidently better in comparison to initial solution.

Optimization of first dam position is not fully reliable. In real, industry solutions, first dam is responsible for turbulence flow zone, while turbulence effects are not taken into account in the present work.

The optimization criteria used in the present work are formulated for the tundish region, in which plug flow is solely excepted. Introduction of the additional criteria for turbulent mixing zone might allow solving the real cases. Verification of the model is only possible on the ground of comparison with operational results of industrial casting tundish. The choice of optimization criteria is dependent on the demands of customer. Any amount of criteria may be involved in the model, which is due to generic multicriterion approach applied. Their precise description and the significance have to be adapted to expectation of customer.

In the present work the commercial FEM codes ADINA-F were utilized in construction of the model of casting tundish. However, the structure of optimization module makes possible the connection with any other commercial codes (e.g. FLUENT) or with the codes prepared individually for specific unit. The method employed in the present work may be also applied to other metallurgical processes, for which reliable numerical models exist.

Acknowledgements

The financial support of AGH - UST project no. 10.10.110.641 is gratefully acknowledged.

REFERENCES

[1] T.B. Harabuchi, R.D. Pehlke, Ciągłe Odlewanie. Urządzenia i Technologie (1988).
[2] L. Zhang, S. Taniguchi, K. Cai, Metallurgical and Materials Transactions B **31B**, 253 (2000).
[3] A.K. Sinha, Y. Sahai, ISIJ Int. **33**, 5 (1993).
[4] J. Falkus, II Int. Conf. Continuous Casting of Steel, Krynica (2004), 51-57.
[5] J. Falkus, Archives of Metallurgy **41** (1996).
[6] J. Jowsa, M. Warzecha, W. Derda, II Int. Conf. Continuous Casting of Steel, Krynica 145-151 (2004).
[7] A. Cwudziński, J. Jowsa, Computer Methods in Materials Science **6**, 123-128 (2006).
[8] R.D. Morales, S. Lopez-Ramirez, J. Palafox-Ramos, D. Zacharias, ISIJ Int. **39** 455-462 (1999).
[9] D.-Y. Sheng, C.-S. Kim, J.-K. Yoon, T.-C. Hsiao, ISIJ Int. **38** 843-851 (1998).
[10] A. Osyczka, Multicriterion Optimization in Engineering. Ellis Horwood Limited, Chichester (1984).

Received: 10 .... 2005.