Heating of materials is energy and costly operations. On those reasons optimization is highly desirable. One of the possible solutions to optimize heating in real time is to use a large number of fast simulations on the basis of them the optimization algorithms have chosen the most appropriate option of the heating control. This solution implies the use of extremely fast but sufficiently accurate simplified mathematical models of heating, the structure and parameters of them are defined based on accurate modelling using computationally intensive but slower classical mathematical-physical models. Based on the operating data of the reheating furnace was build an accurate model of heating. Using the simplified model simulation of heating was done with different heating conditions with downtime during heating. Proposed algorithms including the simulations show that the proposed strategy leads to verifiable savings during heating.

Keywords: heating process, mathematical and physical model, real time control, optimal control
last four zones of furnaces are usually heated, and thus the
direct control of the temperature in the furnace can be done
only within these zones so the first zone is not equipped with
burners.

3. The accurate and simplified models of heating

Algorithms for correcting temperatures in each zone of a
furnace are intended to stabilize the parameters of the heating
charge while material and energy losses are minimized
during heating process. Irregular downtime of charging re-
quires real-time interventions to the control strategy of the
heating. This requires to use algorithms that apply simplified,
fast working models of heating. Based on the rapid multiple
simulations they select the optimal variant of the conduct to
the heating appropriate for the specific situation then.

The parameters of simplified models are obtained by the
application of accurate mathematical models, but their com-
plexity is not able to deliver results in the time required for
real-time control of the heating process [4], [5].

The accurate models of heating

The exact mathematical models of heating works on phy-
sical laws related to the heating of materials and the overall
operation of the furnace. They describe mathematically for-
ulate all physical principles from the fuel consumption through
the combustion conditions, the temperature in each zone of
furnaces to heat transfer to the heated material, heat transfer
inside the heated material and heat loss to the surroundings.

The exact mathematical models have to respect the shape
of real geometric dimensions of the heated material. The mod-
els describing heating of the material in the shape of a cylinder
will used.

The Short cylinder we mean as the intersection of an in-
finite plate of thickness L and an infinite cylinder of radius
R.

We mark variables related to the length (height) coordi-
nate of the cylinder with indexes L and R variables dependent
on the radial coordinate of the cylinder. Then: \( B_{iL} \) is the Biot
criterion relative to the height of the block [1], \( B_{iR} \) the Biot
criterion relative to the radius of the block [1], \( \alpha_2 \) is the overall
heat transfer coefficient [W \( \cdot \) m\(^{-2} \) \( \cdot \) K\(^{-1} \)], \( \tau_i \) is the time constant
of the point inside the cylindrical body [s], \( \tau_R \) is the time
constant of the point inside the cylindrical body [s], \( \tau^R \) is the characteristic
time constant of the cylinder [s], \( \tau^L \) is time constant of the
point inside the plate element [s], \( \tau_1 \) is time constant of the
point on the surface of the plate element [s], \( \tau^1 \) is the character-
istic time constant of the plate element [s], \( \mu^L \) is the \( i^{th} \) root of the characteristic equation plates, \( \mu^R \) is the \( i^{th} \)
root of the characteristic equation plates, \( a \) is the coefficient of
thermal conductivity [m\(^2 \) \( \cdot \) s\(^{-1} \)], \( \lambda \) is the Coefficient of thermal con-
ductivity [W \( \cdot \) m\(^{-1} \) \( \cdot \) K\(^{-1} \)].

The transfer function system the furnace – the point inside
the cylinder with boundary conditions of the third kind can be
described by the equation [9], [10], [11]:

\[
F(x, r, p) = 1 - \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{D^L_{i}D^R_{j} \mu^L_i \mu^R_j}{\mu^L_i \mu^R_j + 1},
\]

where

\[
b^L_i = \left( \frac{\mu^L_i}{\tau^L} \right)^2 \text{[s}^{-1}],
\]

\[
b^R_j = \left( \frac{\mu^R_j}{\tau^R} \right)^2 \text{[s}^{-1}].
\]

The characteristic time constants of the plate and of the cylin-
der are:

\[
\tau^L = \frac{L^2}{a} \text{[s]},
\]

and

\[
\tau^R = \frac{R^2}{a} \text{[s]}.
\]

The roots of the characteristic equations are expressed by
equations:

\[
\mu^L_i = B_{iL} \cot g \mu^L_i,
\]

\[
\mu^R_j \cdot J(\mu^R_j) = B_{iR}.
\]

The gain \( D^L_i \) and \( D^R_i \) are expressed from relations:

\[
D^L_i(x) = \frac{2 \cdot \sin \mu_i \cdot \cos(\mu_i \cdot \frac{x}{L})}{\mu_i + \sin \mu_i \cdot \cos \mu_i},
\]

\[
D^R_i(x) = \frac{2J(\mu_i)J_0(\mu_i \cdot \frac{R}{a})}{\mu_i \cdot [J^2(\mu_i) + J^2_0(\mu_i)]}.
\]

In the transfer function (1) is considered only the first term of
its expansion.

To set the parameters of the model is used the adaptive
identification based on data measurements on real objects of
the reheating furnace.

The model is a resource for setting parameters of simpli-
fied models designed to predictions in real-time and can be
used to check the performance of simplified models.

Simplified models of heating

The aim is to create a sufficiently accurate simplified model
of heating, the implementation of which can carry a large
amount of computation in real time. Models can be based
on many principles. One solution mentioned in the article [4].

These may be formulated by the following mathematic
formulæ:

\[
\tau_1(t) \frac{dT_1(t)}{dt} + T_1(t) = T_f(t)
\]

and

\[
\tau_2(t) \frac{dT_2(t)}{dt} + T_2(t) = T_i(t),
\]

where \( \tau_1(t) \) is outside heat transfer time-variable time constant
[min], \( \tau_2(t) \) is inside heat transfer time-variable time constant
[min], \( T_f(t) \) is temperature of the heated material surface [°C],
\( T_f(t) \) is furnace temperature [°C], \( T_i(t) \) is material heated ma-
terial core temperature [°C].

The model is described by two time-variant linear differ-
etial equations, which affects the external and internal heat
transfer. The model parameters are revised by the results of
the exact mathematical model. The model input is the instantaneous temperature at the furnace, which is calculated from the temperature in each zone furnace [4].

**The forecast**

In our simulated case, we will assume that they are set and stabilized the optimum temperature in each zone of the furnace in case that particular material

If there is a disruption of the regular furnace operation the prediction of parameters of heated material with simulating adjustments the current temperature set point in each zone of the furnace will be executed repeatedly. It is a matter of the optimization algorithms to find the optimal variant of heating from all predicted ones [7], [8].

**Optimization**

The aim of the optimization algorithm is to stabilize the monitored parameters of heated material on the outlet of the furnace. The straightforwardness of simplified real-time models allows observing simply the value of optimality in the form [12]:

$$ML(t) = 60 \times \sqrt{\frac{76233 \times 2}{T_s(t) \times e^{\left(t / 17057\right)}}},$$

where $ML(t)$ is the metal loss [kg/m$^2$], $T_s(t)$ is the temperature of the heated material surface [$^\circ$C].

The criterion allows the using optimization algorithms to minimize the costs associated with the iron lost. This additional criterion cannot directly include to optimization algorithms that utilize fast simplified models [13], [14], [15].

The general principle of optimization algorithms of the temperature correction in each furnace zone is a gradual temperature decrease in controlled zones of the furnace so that temperatures would be increasing gradually again to the standard values when standard continuous mode of charging (after downtime) starts again and in the same time it would be guaranteed for the individual rows of heated material in the furnace a minimal deviation to the technologically references of parameters of a hot material when putting off form the furnace.

The actions of temperature set points in each furnace zone must be such that they respect the technological requirements for the maximum allowable decrease and increase of temperatures in the furnace and the temperatures in each zone do not exceed the specified limits of technology simultaneously.

For own optimization it may be successfully used the methods of artificial intelligence in addition to traditional methods, such as genetic algorithms, etc. [13] [16], [17].

4. **The results of the simulation of optimization of heating**

**Correcting the temperature in the furnace zones**

As an illustrative example of the temperature correction in each zone the furnace the heating with an interval of one step in the duration of four minutes was simulated, which was suspended in 160 minutes by downtime in the duration of one hour. After this idle again followed regular progression of heated material in the furnace with a period of 4 minutes.

The simulation results can be seen in Figure 2, in which are plotted as curves of monitored values without correction of the temperature of zones in the furnace (solid line) as with the correction (dashed line).

![Fig. 1. The simulation results of heating – the time course of the monitored quantities](image-url)

**The simulation results**

The simulation results without correction of the temperature in each zone furnace as with the correction of heating to optimize it can be seen in Figures 3. It is complicated to correct mean temperature and it is only possible through the adjustment of the surface temperature.

**The simulation results without correction:**

The surface temperature and the mean temperature of the heated material during heating since the downtime till pulling out from the furnace are too high (Mean temp = 1283°C, Surf. temp = 1289°C).

This leads to an increase in the metal loss during the heating compared to the standard heating without downtime (101.5 kg/m$^2$).

It leads to increasing of the energy needs associated with prolonged heating compared to the standard heating (2724.813 kWs/kg).

**The simulation results with correction:**

The surface temperature and the mean temperature of the heated material does not rise so sharply since the moment of downtime till pulling out of the oven already during the heating and in the time of discharging these temperatures are
This leads to a reduction metal losses during heating compared to the heating with downtime and without a correction (92.1 kg/m² which is about 10%) and there is slight reduction in energy needs of the heating compared to the heating with downtime and without correction (the decrease is down to 2701.488 kWs/kg, what means about 1% of energy savings).

5. Summary

The aim of the heating process optimizing using the set point corrections of control variables of control loops of each zone of the reheating furnace is to stabilize the parameters of heated material at the furnace outlet while minimizing energy and material demands of heating.

For creating fast and simple models for self-optimization artificial intelligence can also be successfully applied, such as neural networks and principles of evolutionary and genetic algorithms.

Two simulations were performed to compare this with the heating without downtime. The both simulations with downtime show an increasing of a surface and a mean temperature of the heated material at leaving the furnace. The surface temperature with the correction of heating was much closer to the case without delay, but this temperature can only indirectly affect by changes in a surface temperature of the heated material.

The consumption increased and also the metal loss was higher when the downtime when correcting heating lower values have been reported compared to the case without correction. Even if there consumption and metal loss were not percentage too important for energy and material demands of production significant decreases in financial savings would be reached.

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