1. Introduction

Finding innovative ways to improve the physical properties of non-ferrous metal alloys is mainly based on experimental studies. This situation leads to a lack of mathematical models that would describe changes taking place in metal when it is subjected to different treatments improving its properties. As a result of this situation, of great importance is to develop a methodology for the construction of heuristic models that would allow, be it even in a very approximate way, finding, a relationship between the physical parameters of an alloy and the applied variants of a treatment improving these properties. It should be noted that the main difficulty in creating this class of models results from some limitations associated with the physical experiments which involve costs, and also with the complexity of the conducted studies. The consequence is that, on the one hand, the aim is to test as large number of the variants of the toughening treatment as possible, while, on the other, the idea is to reduce the number of samples tested. In this situated, it is easy to conclude that the use of traditional statistical methods would be completely senseless. Therefore, it seems reasonable to seek opportunities for the construction of heuristic models, which by their very nature are of an approximate character, but owing to this can base on a very limited experimental material [1,2,3,4,5, 9, 10]. The paper proposes this type of an approach using classification trees and fuzzy logic rules.

2. Experimental results

The examined experiments referred to the effect of modification and toughening (quenching and tempering) of the BA1044 alloy on its mechanical properties, determined by the following parameters: yield strength – \( R_{p0.2} \), tensile strength – \( R_m \), elongation – \( A \). Measurements of these values were taken on 72 samples subjected to different variants of the toughening treatment. Fragment of the obtained results are shown in Table 1, using the following symbols: without modification – L, modified with Ca ++ C – M, modified with CaC _2 – N, modified with Ti ++ C _2 – P, modified with Zr ++ B – S, modified with Mischmetall – T, microjet quenching – (P), as cast condition (without treatment) – (L). Tested were 24 variants of the toughening treatment, and taking into account that some samples were unsuccessful, 2-3 measurements fell to each variant. Under these conditions, the hypothetical model of relationships between parameters describing the conducted studies included 24 input quantities and 5 output quantities. So, the use of the traditional modelling methods would be under such circumstances out of the question.
### TABLE 1

<table>
<thead>
<tr>
<th>Modified</th>
<th>Quenching</th>
<th>Tempering</th>
<th>Rm</th>
<th>Rp0,2</th>
<th>A5</th>
<th>Toughening treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L</td>
<td>brak</td>
<td>678</td>
<td>287</td>
<td>5</td>
<td>L</td>
</tr>
<tr>
<td>2</td>
<td>L</td>
<td>brak</td>
<td>678</td>
<td>273</td>
<td>6</td>
<td>L</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>brak</td>
<td>672</td>
<td>285</td>
<td>4</td>
<td>L</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>P</td>
<td>822</td>
<td>225</td>
<td>4</td>
<td>P</td>
</tr>
<tr>
<td>5</td>
<td>L</td>
<td>P</td>
<td>802</td>
<td>296</td>
<td>5</td>
<td>P</td>
</tr>
<tr>
<td>6</td>
<td>L</td>
<td>P</td>
<td>825</td>
<td>345</td>
<td>3</td>
<td>P</td>
</tr>
<tr>
<td>7</td>
<td>L</td>
<td>P</td>
<td>S1</td>
<td>817</td>
<td>389</td>
<td>2</td>
</tr>
</tbody>
</table>

### 3. Heuristics and heuristic models

The term “heuristic” means certain orbitally (intuitively) selected quantity, which can characterise the tested process (phenomenon) under the conditions when creating its precise description is not possible due to the lack of sufficient knowledge, or it is too difficult (e.g. on account of the excessive computational complexity). Currently, the use of heuristics is becoming increasingly common in various areas of research. Interesting use of heuristics to create models of different processes can be found, among others, in [6], where the heuristics were used for traffic modelling, or in [7] which presents a heuristic approach to the semantic classification of text documents. With the use of the adopted heuristics it is possible to construct a model of a process, bearing in mind the fact that dependencies (relationships) obtained in this way can describe this model in an approximate way only. Therefore, whenever possible, the heuristic model should be verified for a reliability of its performance. Both the selection of heuristics as well as the interpretation of the results of the model application are the tasks of the knowledge engineer / user. The procedure outlined above is presented in a pictorial way in Figure 1.

![Fig. 1. Schematic diagram methodology for heuristic model construction](image)

From a formal point of view, the operation of a heuristic model can be interpreted as a sequence of mappings:

$$H : X \rightarrow U; M : U \rightarrow V; G : V \rightarrow Y$$

where:

- $H$ – operator mapping the space of physical parameters into the space of heuristics,
- $M$ – operator mapping the input heuristics of a model into the output heuristics,
- $G$ – operator mapping the output heuristics of a model into the space of real parameters (physical). It should be noted that operators $H, M, G$ do not always take the standard form of functional dependencies, but may involve complex calculation procedures. In fact, the freedom of choice of the used transformations is an important feature of this class of models. Below two variants of models used for the interpretation of the results of the experimental studies described in the previous section will be presented. In both cases, the discussion has been limited to selected fragments of the data only to illustrate the same methodology of studies.

### 4. Classification trees

In a given case, the heuristic model ($M$) allows us to generate a classification tree, specifying the sequences of actions (taken in the considered process of toughening treatment) that lead to classification of the selected output parameter as belonging to a group with the specified properties.

To generate the classification trees, Statistica package was used, and the tree structure was preceded by a statistical pre-treatment to extract the parameters of major importance. This stage can be considered a selection of heuristic, which in this case will comprise the most important toughening operations (operator $H$).

The selected set of input parameters was used as a basis to generate the classification trees for all the variables ($R_m$, $R_{p0.2}$, $A5$). The tree obtained for $R_m$ is shown in Figure 2. For each final leaf of the tree, one can specify a set of actions corresponding to the obtained group of results with a certain mean and a variance. Since the aim is to obtain the highest possible value of $R_m$, attention deserve the leaves with a mean of 830 and 816, but it should be noted that in the first case the variance is much greater.

![Fig. 2. Classification Tree for Rm](image)

In this particular case, the last step in the interpretation of the model as a classification tree (operator $G$) will consist in reading from this tree a set of actions needed to achieve a given group of results.

As we can see this procedure gives us certain idea about the process of toughening, but as it relates to each parameter...
separately (separate classification trees), it is not fully satisfactory.

5. Fuzzy models

Another type of a heuristic model that has been constructed for the interpretation of the measurement data under consideration is a model based on the use of fuzzy logic [8]. Here the experience gained in creating classification trees has been used, and owing to this it was possible to reduce the number of input variables to three, which are defined as follows:

- modifier – a variable representing all the usually applied methods of modification,
- quenching – a variable indicating the use of quenching or lack thereof,
- tempering – a variable specifying the applied variant of tempering: no tempering, tempering of type S1, or tempering of type S2.

As output variables the following parameters have been accepted: \( R_m \) – tensile strength, \( R_{\sigma,0.2} \) – yield strength, \( A_5 \) – permanent elongation. With such an aggregation of variables, it becomes possible to construct a model, which will characterise in a comprehensive manner the relationships that occur in the investigated process of toughening.

The adoption of a model based on fuzzy logic naturally determines the form of heuristics that represent individual variables (input and output) – these will be appropriately selected functions of membership to preset areas where the value of a given variable occurs. In the case of the input variables referred to above, these areas correspond to the applied variants of toughening (mutually exclusive), and therefore the membership function of a trapezoidal type has been adopted here. The same run is observed in functions plotted for the variables quenching and tempering. In this way the development of heuristics for the input variables has been completed (Fig. 1).

The heuristics of output variables are also membership functions, but the way they are defined will be slightly different due to the continuous nature of these variables. Here, to the linguistic concepts applied in fuzzy logic (e.g. small, medium, large) have been assigned certain ranges of values of the variable by using a function of the Gaussian type. The run of such functions for the case of the variable \( R_m \) is shown in Figure 4. Similar character is observed in functions adopted for the variables \( R_{\sigma,0.2} \) and \( A_5 \).

The construction of the model (M in Figure 1) consists in making rules representing the relationships between input and output variables. Using the results obtained in the construction of classification trees, fifty nine rules that define the operation of the model have been formulated.

Below there are some selected rules for the variable \( R_m \): From the above rules it follows that modification does not have a significant impact on the value of \( R_m \), on the other hand, of great importance are quenching and tempering.

Full picture of the correlation between input and output measurable values that is obtained as soon as all the rules are introduced to the requesting system, which determines the degree of membership to the highlighted areas of output variables. In the case under consideration, the model (M) has been implemented using Matlab fuzzy logic system.

The degrees of membership obtained from the requesting system make a heuristic representation of the physical parameters of the metal. To obtain the values of these parameters, a defuzzification should be performed (which corresponds to the operation G in Figure 1).

**TABLE 2**

<table>
<thead>
<tr>
<th>Rules representing the relationships</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{modifier} = N ) &amp; ( \text{Quenching} = 1 ) &amp; ( \text{tempering} = \text{none} ) ( \rightarrow ) ( R_m = 725 )</td>
</tr>
<tr>
<td>( \text{modifier} = S ) &amp; ( \text{Quenching} = 1 ) &amp; ( \text{tempering} = \text{none} ) ( \rightarrow ) ( R_m = 725 )</td>
</tr>
<tr>
<td>( \text{modifier} = S ) &amp; ( \text{Quenching} = 1 ) &amp; ( \text{tempering} = S1 ) ( \rightarrow ) ( R_m = 816 )</td>
</tr>
<tr>
<td>( \text{modifier} = N ) &amp; ( \text{Quenching} = 1 ) &amp; ( \text{tempering} = S1 ) ( \rightarrow ) ( R_m = 816 )</td>
</tr>
<tr>
<td>( \text{modifier} = L ) &amp; ( \text{Quenching} = 1 ) &amp; ( \text{tempering} = S2 ) ( \rightarrow ) ( R_m = 739 )</td>
</tr>
</tbody>
</table>

MatLab system performs this operation by setting the, so-called, centre of gravity of the solid obtained after adding the appropriate degrees of membership, but a more expressive form of representation of the results obtained from the model are 3D charts illustrating the relationships between the three selected parameters. A graph showing the relationship between \( R_m \) and operations such as quenching and tempering is shown in Figure 5. Interpreting this graph it can be concluded that the highest value of \( R_m \) is obtained by quenching (conventional scale of 0.6-1) and tempering in type S1 (conventional scale of 0-0.8).

Fig. 3. Membership function for the variable \( R_m \)

Fig. 4. Relationship between \( R_m \) and operations quenching and tempering
6. Final conclusions

This paper presents two types of heuristic models for the toughening process of BA1044 alloy, based on the results of the physical experiments. In both cases, the results of modelling give only a very rough idea about the relationships that occur in the real process, but in view of the small number of measurement data, the result of the investigations obtained in this way should be considered satisfactory.

Due to the limited volume of this study, the idea of a broader interpretation of the numerical results was given up, focusing attention on showing the same methodology for the creation of heuristic models that can be used in a variety of cases where the limited field for experiments on the real process makes building of models describing this process in a more precise way impossible.

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REFERENCES


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