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**INACCURACY IN SELF-SYNCHRONISATION OF VIBRATORS OF TWO-DRIVE VIBRATORY MACHINES CAUSED BY INSUFFICIENT STIFFNESS OF VIBRATORS MOUNTING**

**ZAKŁÓCENIA SAMOSYNCHRONIZACJI WIBRATORÓW DWUNAPĘDOWYCH MASZYN WIBRACYJNYCH NA SKUTEK NIEPEŁNEJ SZTYWNOŚCI POSADOWIENIA WIBRATORÓW**

The influence of an insufficient stiffness of inertial vibrators mounting to a vibratory machine body on phase angles of unbalanced masses and on a machine motion was analysed in the paper.

The mathematical model of the effect was developed and the analytical dependencies determining the influence of the mounting elasticity on the synchronisation and machine movement – useful in engineering practice – were obtained.

*Keywords:* vibratory machines, self-synchronisation errors

W pracy poddano analizie wpływ nie w pełni sztywnego zamocowania wibratorów inercyjnych do korpusu maszyn wibracyjnych na kąty fazowe mas niewyważonych i ruch maszyny.

Zbudowano model matematyczny zjawiska i uzyskano zależności analityczne określające wpływ podatności zamocowania na przebieg synchronizacji i ruch maszyny, przydatne w praktyce inżynierskiej.

**1. Introduction**

There are situations, met in industrial practice, that two-drive vibratory machines utilising the self-synchronisation of vibrators, are not properly operating regardless of fulfilling all conditions stipulated in the references [1]. Passing over the influence of the asymmetry of drive or influence of collisions with a material feed to synchronization [2,3,4], the insufficient stiffness of vibrators mounting, found out in industrial tests [5], can be considered one of the essential reasons of such situation. As it was shown in the last quoted paper, the insufficient stiffness of vibrators mounting can lead to a substantial change of the synchronisation phase angle and due to that, to a completely wrong machine work.

Since this effect was not theoretically investigated, **the aim of the hereby paper is developing of the mathematical model of this effect and the derivation of computational dependencies allowing to estimate the necessary stiffness of vibrators mounting.**

In order to do that the diagram of two-vibrator over-resonance machine, shown in Fig. 1, will be considered.

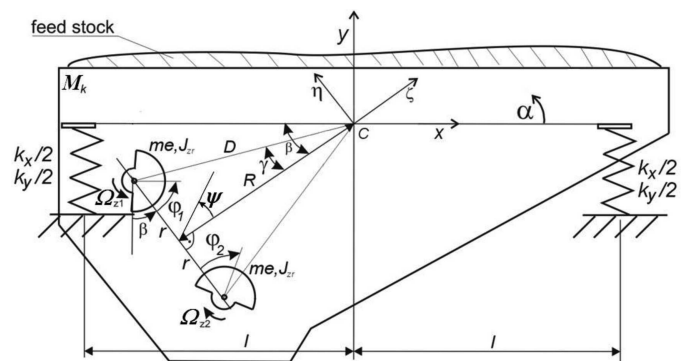


Fig. 1. Two-vibrator over-resonance vibratory machine calculation model

where:

$m$  – unbalanced mass [kg],

$e$  – eccentric of a rotor unbalance [m],

$M_k$  – machine body mass [kg],

$M = M_k + 2m$  – mass of the vibrating machine part (body and vibrator) [kg],

$C$  – mass centre of the body with unbalanced masses brought to the vibrator axis of rotation

$J$  – central moment of the body inertia with unbalanced masses brought to the vibrator axis of rotation [kgm<sup>2</sup>],

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$J_{zr}$  – moment of inertia of the vibrator together with engine, calculated versus the axis of rotation [kgm<sup>2</sup>],  
 $k_x, k_y, k_\xi, k_\eta$  – coefficients of elasticity of the body supporting system along axes:  $x$ ,  $y$  and  $\xi$ ,  $\eta$  [N/m]– correspondingly,

$\Omega_{z1}, \Omega_{z2}$  – moments exerted to the rotating masses originated from the drive and resistance to motion [Nm],

The problem formulated above cannot be solved directly by classic methods (e.g. by the Blechman's integral criterion [1]), since an elastic connection of vibrators with the machine body significantly widens the problem and in consequence does not allow to obtain the analytical solution in a form useful in an engineering practice.

In order to solve this problem the *averaging method* [6], in which the advantage was taken of the fact that quantitative connections occurring for real systems enable taking two essential simplifying assumptions, was applied in the paper.

<sup>10</sup> It was assumed, that deformations in places of vibrators mounting are of a quasi-static character, it means that the deformation value is proportional and cophasal to the force transmitted by the vibrator. The correctness of this assumption can be shown on the basis of the following estimation:

- for typical machines, softly elastically placed and vibrating with an amplitude  $A$  and frequency  $\omega$ , the combined excitation force of  $n$  vibrators, each of a force  $P$ , imparts to the machine of a mobile mass  $M$ , the amplitude of acceleration:

$$\frac{nP}{M} \approx A\omega^2 \quad (1)$$

- frequency  $\omega_\psi$  of natural vibrations of the vibrator on its elastic mounting - equals:

$$\omega_\psi = \sqrt{\frac{k_\psi}{m_w}} \quad (2)$$

where  $m_w$  denotes the mass of the vibrator together with its casing, while  $k_\psi$  is the coefficient of elasticity of its mounting.

If it is assumed that the vibrator mounting deformation – under an influence of its excitation force – should not exceed the machine vibration amplitude  $A$ , i.e.

$$\frac{P}{k_\psi} \leq A \quad (3)$$

and the excitation force  $P$  is approximated in accordance with the expression (1), then according to (2), for  $k_\psi \geq P/A$  (3), the following dependence is obtained:

$$\omega_\psi \geq \sqrt{\frac{M\omega^2}{n} \cdot \frac{1}{m_w}} = \omega \sqrt{\frac{M}{n \cdot m_w}} \quad (4)$$

Since the machine mass  $M$  is significantly larger than the total mass of vibrators  $n \cdot m_w$ , the vibrator frequency  $\omega_\psi$  on its support is significantly larger than the working frequency  $\omega$ .

This means the under-resonance operation and allows assuming a quasi-static character of deformations of vibrators mounting elements.

<sup>20</sup> It was assumed that the component originated from the Coriolis acceleration can be omitted at the determination of the vibrators axles acceleration.

- Acceleration of the axle of the  $i^{th}$  vibrator (Fig.1) is written in a form:

$$\bar{a}_i = \bar{a}_{iu} + \bar{a}_{iw} + \bar{a}_{icor} \quad (5)$$

where individual terms mean:

$\bar{a}_i$ – absolute acceleration of the  $i^{th}$  vibrator axle,

$\bar{a}_{iu}$ – acceleration of transportation, resulting from the translatory and angular motion of the machine,

$\bar{a}_{iw}$ – relative acceleration, resulting from the vibrator axle movement in the direction  $\psi$  determined by the elasticity direction of the vibrator mounting system,

$\bar{a}_{icor} = 2\bar{\omega}_u \times \bar{v}_{iw}$ – Coriolis acceleration.

The order of the tangential component (normal component = 0) of the relative acceleration in vibrating motion with the amplitude  $A_\psi$  and frequency  $\omega$

$$|\bar{a}_{iw}|_{\max} = A_\psi \omega^2$$

is now compared with the order of the Coriolis acceleration:

$$|\bar{a}_{icor}|_{\max} = 2\omega_u \cdot A_\psi \omega \cdot \sin(\bar{\omega}_u, \bar{v}_{iw})$$

Since the transportation angular frequency  $\omega_u$  is several times smaller (if it exists at all) than the working frequency  $\omega$ , which results from the dimensional ratios and linear vibrations amplitudes of vibratory machines, the following occurs:  $a_{icor} \ll a_{iw}$ . This allows to omit the Coriolis acceleration in a further analysis.

## 2. Mathematical analysis of the self-synchronisation effect in the system of an insufficient mounting stiffness of individual vibrators

As the first analysed case the system, in which each of vibrators is placed independently will be considered. The stiffness of the  $i^{th}$  ( $i=1,2$ ) vibrator mounting to the body is insufficient, but characterised by the possibility of shifting the axle along the direction defined by angle  $\psi$  (Fig.1), resulting from the limited elasticity  $k_\psi$  in this direction.

In accordance with the idea of the averaging method, the equations of motion of the machine body were written with distinguishing slow- and fast-changing effects.

Thus, assuming the equality of angular velocities of both vibrators  $\dot{\phi}_1 = \dot{\phi}_2$  and their 'slow-changes' ( $\omega \approx \text{const}$ ), it is possible to write the equation of motion of the body in the absolute system  $\xi, \eta$ , in a first approximation form:

$$M\ddot{\xi} + k_\xi \xi = me\omega^2[\sin \phi_1 + \sin \phi_2] \quad (6a)$$

$$M\ddot{\eta} + k_\eta \eta = me\omega^2[\cos \phi_2 - \cos \phi_1] \quad (6b)$$

$$J\ddot{\alpha} + k_y l^2 \alpha = me\omega^2 r[\sin \phi_2 - \sin \phi_1] + me\omega^2 R[\cos \phi_1 - \cos \phi_2] \quad (6c)$$

where:  $\xi, \eta$  – absolute coordinates determining the position of the body mass centre,

$\alpha$  – angle of rotation of the body,

$$k_\xi = k_x \cos^2 \beta + k_y \sin^2 \beta$$

$$k_\eta = k_y \cos^2 \beta + k_x \sin^2 \beta,$$

- the remaining notations are the same as in Fig. 1.

Denoting  $\phi_1 - \phi_2 = \Delta\phi = \text{const}$  and assuming  $\phi_2 = \omega t$ , equations (6a, 6b, 6c), after rearrangements, can be presented in a form:

$$M\ddot{\xi} + k_\xi \xi = me\omega^2 \sqrt{2(1 + \cos \Delta\phi)} \sin(\omega t + \delta) \quad (7a)$$

$$\text{where: } \sin \delta = \frac{\sin \Delta\phi}{\sqrt{2(1 + \cos \Delta\phi)}}, \quad \cos \delta = \frac{1 + \cos \Delta\phi}{\sqrt{2(1 + \cos \Delta\phi)}}$$

$$M\ddot{\eta} + k_\eta \eta = me\omega^2 \sqrt{2(1 - \cos \Delta\phi)} \sin(\omega t + \kappa) \quad (7b)$$

$$\text{where: } \sin \kappa = \frac{1 - \cos \Delta\phi}{\sqrt{2(1 - \cos \Delta\phi)}}, \quad \cos \kappa = \frac{\sin \Delta\phi}{\sqrt{2(1 - \cos \Delta\phi)}}$$

$$J\ddot{\alpha} + k_y l^2 \alpha = me\omega^2 D \sqrt{2(1 - \cos \Delta\phi)} \sin(\omega t + \lambda) \quad (7c)$$

$$\text{where: } \sin \lambda = \frac{R(\cos \Delta\phi - 1) - r \sin \Delta\phi}{D \sqrt{2(1 - \cos \Delta\phi)}}, \quad \cos \lambda = \frac{r(1 - \cos \Delta\phi) - R \sin \Delta\phi}{D \sqrt{2(1 - \cos \Delta\phi)}}$$

Particular integrals of these equations, describing the steady state, are of a form:

$$\xi(t) = \frac{me\omega^2 \sqrt{2(1 + \cos \Delta\phi)}}{k_\xi - M\omega^2} \sin(\omega t + \delta) \quad (8a)$$

$$\eta(t) = \frac{me\omega^2 \sqrt{2(1 - \cos \Delta\phi)}}{k_\eta - M\omega^2} \sin(\omega t + \kappa) \quad (8b)$$

$$\alpha(t) = \frac{me\omega^2 D \sqrt{2(1 - \cos \Delta\phi)}}{k_y l^2 - J\omega^2} \sin(\omega t + \lambda) \quad (8c)$$

The analysed, up to the present, dynamic equations were describing the body vibrations at the assumption that the vibrators angular motion for the steady state can be considered the uniform motion. Such assumption is tantamount to omitting the body vibration influence on the vibrators running. Presently the vibrators equations of

motion will be written with taking into account those couplings, it means in the non-inertial system related to the machine body executing the vibrating motion described above.

When the moments from forces of the transportation are applied to vibrators, their angular motion equations are in a form:

$$J_{zr} \ddot{\phi}_1 = \Omega_{z1} - me\ddot{\xi}_1 \cos \phi_1 - me\dot{\eta}_1 \sin \phi_1 \quad (9a)$$

$$J_{zr} \ddot{\phi}_2 = \Omega_{z2} - me\ddot{\xi}_2 \cos \phi_2 + me\dot{\eta}_2 \sin \phi_2 \quad (9b)$$

where :

$\Omega_{z1,2}$  – external moments (difference between the driving and anti-torque moments),

$J_{zr}$  – moment of inertia of the vibrator together with engine, calculated versus the axis of rotation.

**Vibratory moments**  $\Omega_{wi}$ ,  $i=1,2$ , are expressed as:

$$\Omega_{w1} = -me(\ddot{\xi}_1 \cos \phi_1 + \dot{\eta}_1 \sin \phi_1) \quad (10a)$$

$$\Omega_{w2} = -me(\ddot{\xi}_2 \cos \phi_2 - \dot{\eta}_2 \sin \phi_2) \quad (10b)$$

On the bases of the previously derived solutions of the body motion (not taking into account influences of vibratory moments on vibrators running) the acceleration components of both vibrators axes will be determined. The acceleration of axes of individual vibrators, after taking into account the transportation and the relative motion resulting from the local elasticity as well as after omitting expressions of a lower order versus the remaining ones, has in the absolute system  $\xi\eta$  the following components:

$$\ddot{\xi}_1 \approx \ddot{\xi} - \ddot{\alpha} \cdot r - \frac{me\omega^4}{k_\psi} \cos \psi \cdot \sin(\omega t + \Delta\phi - \psi) \quad (11a)$$

$$\dot{\eta}_1 \approx \dot{\eta} - \dot{\alpha} \cdot R - \frac{me\omega^4}{k_\psi} \sin \psi \cdot \sin(\omega t + \Delta\phi - \psi) \quad (11b)$$

$$\ddot{\xi}_2 \approx \ddot{\xi} + \ddot{\alpha} \cdot r - \frac{me\omega^4}{k_\psi} \cos \psi \cdot \sin(\omega t - \psi) \quad (11c)$$

$$\dot{\eta}_2 \approx \dot{\eta} - \dot{\alpha} R + \frac{me\omega^4}{k_\psi} \sin \psi \cdot \sin(\omega t - \psi) \quad (11d)$$

After substituting in Eq. (11) the second derivatives of Eq. (8) and introducing the obtained values into Eq. (10), it is possible to calculate the vibratory moment:

$$\begin{aligned} \Omega_{w1} = & -\sqrt{2}m^2e^2\omega^4 \left[ \frac{\sqrt{1 + \cos \Delta\phi}}{M\omega^2 - k_\xi} \sin(\omega t + \delta) \cdot \cos(\omega t + \Delta\phi) - \right. \\ & \frac{rD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda) \cdot \cos(\omega t + \Delta\phi) + \\ & \frac{\sqrt{1 - \cos \Delta\phi}}{M\omega^2 - k_\eta} \sin(\omega t + \kappa) \cdot \sin(\omega t + \Delta\phi) - \\ & \left. \frac{RD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda) \cdot \sin(\omega t + \Delta\phi) \right] + \end{aligned}$$

$$\frac{m^2 e^2 \omega^4}{k_\psi} [\cos \psi \cdot \sin(\omega t + \Delta\phi - \psi) \cdot \cos(\omega t + \Delta\phi) + \sin \psi \cdot \sin(\omega t + \Delta\phi - \psi) \cdot \sin(\omega t + \Delta\phi)] \quad (12)$$

The averaged value of the vibratory moment  $\tilde{\Omega}_{w1}(t)$  for the vibration period  $T=2\pi/\omega$  equals :

$$\begin{aligned} \tilde{\Omega}_{w1} = & \frac{-\sqrt{2}m^2 e^2 \omega^5}{2\pi} \left[ \frac{\sqrt{1 + \cos \Delta\phi}}{M\omega^2 - k_\xi} \int_0^{2\pi/\omega} \sin(\omega t + \delta) \cdot \right. \\ & \left. \cos(\omega t + \Delta\phi) dt - \frac{rD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \lambda) \cdot \cos(\omega t + \Delta\phi) dt + \right. \\ & \left. \frac{\sqrt{1 - \cos \Delta\phi}}{M\omega^2 - k_\eta} \int_0^{2\pi/\omega} \sin(\omega t + \kappa) \cdot \sin(\omega t + \Delta\phi) dt - \right. \\ & \left. \frac{RD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \lambda) \cdot \sin(\omega t + \Delta\phi) dt + \right. \\ & \left. \frac{m^2 e^2 \omega^4}{k_\psi} \cdot \frac{\omega}{2\pi} [\cos \psi \cdot \int_0^{2\pi/\omega} \sin(\omega t + \Delta\phi - \psi) \cdot \cos(\omega t + \Delta\phi) dt + \sin \psi \cdot \right. \\ & \left. \int_0^{2\pi/\omega} \sin(\omega t + \Delta\phi - \psi) \cdot \sin(\omega t + \Delta\phi) dt] \right] \quad (13) \end{aligned}$$

After calculating the integrals and using the dependence (7) (determining the values of angles  $\lambda, \kappa, \delta$ ), it is possible to obtain, after rearrangements, the following equation:

$$\tilde{\Omega}_{w1} = \frac{-m^2 e^2 \omega^4}{2} \left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} \right] \sin \Delta\phi \quad (14)$$

Performing the similar procedure versus the vibratory moment of the second vibrator the following is obtained:

$$\tilde{\Omega}_{w2} = \frac{m^2 e^2 \omega^4}{2} \left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} \right] \sin \Delta\phi \quad (15)$$

As can be seen, the form of equations (14) and (15) is identical as in the case of the vibrators stiffly placed [2,3]. The similar result can be obtained for vibrators

elastically placed in two mutually perpendicular directions.

This means, that an insufficient stiffness of individual vibrators mounting does not influence the character of their self-synchronisation.

### 3. The case of the combined placement of the vibrators set on the supporting system of insufficient stiffness in one or two directions

The case when the system of two stiffly connected vibrators is elastically mounted in the direction  $\psi$  (Fig.1), with the coefficient of elasticity  $k_\psi$ , is considered.

In this case the combined acceleration of vibrators axes, originated from the transportation motion and local deformations, obtains (after omitting components of a lower order than the remaining ones) the following form:

$$\ddot{\xi}_1 \approx \ddot{\xi} - \ddot{\alpha} \cdot r - \frac{m e \omega^4}{k_\psi} \cos \psi \cdot [\sin(\omega t + \Delta\phi - \psi) + \sin(\omega t + \psi)] \quad (16a)$$

$$\ddot{\eta}_1 \approx \ddot{\eta} - \ddot{\alpha} \cdot R - \frac{m e \omega^4}{k_\psi} \sin \psi \cdot [\sin(\omega t + \Delta\phi - \psi) + \sin(\omega t + \psi)] \quad (16b)$$

$$\ddot{\xi}_2 \approx \ddot{\xi} + \ddot{\alpha} \cdot r - \frac{m e \omega^4}{k_\psi} \cos \psi \cdot [\sin(\omega t + \Delta\phi - \psi) + \sin(\omega t + \psi)] \quad (16c)$$

$$\ddot{\eta}_2 \approx \ddot{\eta} - \ddot{\alpha} \cdot R - \frac{m e \omega^4}{k_\psi} \sin \psi \cdot [\sin(\omega t + \Delta\phi - \psi) + \sin(\omega t + \psi)] \quad (16d)$$

Calculating, in a similar fashion as before, the current as well as the averaged – for the vibration period – values of vibratory moments applied to vibrators 1 and 2 it is possible to obtain:

$$\tilde{\Omega}_{w1} = \frac{-m^2 e^2 \omega^4}{2} \left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} \right] \sin \Delta\phi - \frac{m^2 e^2 \omega^4}{2k_\psi} \cdot \sin(\Delta\phi - 2\psi) \quad (17)$$

$$\tilde{\Omega}_{w2} = \frac{m^2 e^2 \omega^4}{2} \left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} \right] \sin \Delta\phi + \frac{m^2 e^2 \omega^4}{2k_\psi} \cdot \sin(\Delta\phi - 2\psi) \quad (18)$$

As it results from the form of equations (17) and (18), a partially elastic placement of the vibrators set in the direction  $\psi$  changes the conditions of vibrators synchronisation. Thus, e.g. in case of a relatively soft placing the second member of both dependencies can decide on the synchronisation character imposing the condition of the solution stability  $\Delta\varphi = 2\psi$  and e.g. for angle  $\psi = \pi/2$

can lead to the counter synchronisation  $\Delta\varphi = \pi$ , causing a stoppage of the machine working motion and initiating its movement in the perpendicular direction.

The stationary motion form can be determined by solving, on account of the phase angle  $\Delta\varphi$ , equation:  $\tilde{\Omega}_{w2} - \tilde{\Omega}_{w1} = 0$ , it means:

$$\left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} \right] \sin \Delta\phi + \frac{1}{k_\psi} \cdot \sin(\Delta\phi - 2\psi) = 0 \quad (19)$$

When the vibrators set – apart from the elasticity of mounting in the direction  $\psi$  – has the identical elasticity in the direction  $\psi + \pi/2$  equation (19) obtains the identical form as for the stiff mounting, which means that the set is not sensitive to this type of elasticity.

#### 4. Combined placement of the vibrators set exhibiting the rotational elasticity

The case of the vibrators set placement which allows for small angular movements around the mid-point of a line segment joining vibrators axles, will be considered. The rotational elasticity occurring in this case, understood as the ratio of the moment to the angle of rotation, is denoted as  $k_o$  [Nm]. If, as for systems of a linear elasticity, the high, in respect to working frequency, tuning of the system: vibrators set - its supporting system, is assumed, the components of the vibrators axes acceleration (after taking into account the influence of elastic rotations and omitting expressions of significantly lower values than the remaining ones) can be written in the following form:

$$\ddot{\xi}_1 \approx \ddot{\xi} - \ddot{\alpha} \cdot r + \frac{me\omega^4 r^2}{k_o} [\sin \omega t - \sin(\omega t + \Delta\phi)] \quad (20a)$$

$$\ddot{\eta}_1 \approx \ddot{\eta} - \ddot{\alpha} \cdot R \quad (20b)$$

$$\ddot{\xi}_2 \approx \ddot{\xi} + \ddot{\alpha} \cdot r - \frac{me\omega^4 r^2}{k_o} [\sin \omega t - \sin(\omega t + \Delta\phi)] \quad (20c)$$

$$\ddot{\eta}_2 \approx \ddot{\eta} - \ddot{\alpha} R \quad (20d)$$

When the vibratory moment  $\Omega_{w1}$  is calculated according to (10a):

$$\Omega_{w1} = -\sqrt{2}m^2 e^2 \omega^4 \left[ \frac{\sqrt{1 + \cos \Delta\phi}}{M\omega^2 - k_\xi} \sin(\omega t + \delta) \cdot \cos(\omega t + \Delta\phi) - \frac{rD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda) \cdot \cos(\omega t + \Delta\phi) + \frac{\sqrt{1 - \cos \Delta\phi}}{M\omega^2 - k_\eta} \sin(\omega t + \kappa) \cdot \sin(\omega t + \Delta\phi) - \right]$$

$$\frac{RD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \sin(\omega t + \lambda) \cdot \sin(\omega t + \Delta\phi)] - \frac{m^2 e^2 \omega^4 r^2}{k_o} [\sin \omega t - \sin(\omega t + \Delta\phi)] \cos(\omega t + \Delta\phi) \quad (21)$$

After averaging equation (21) for the period of vibrations, the following is obtained:

$$\tilde{\Omega}_{w1} = \frac{-\sqrt{2}m^2 e^2 \omega^5}{2\pi} \left[ \frac{\sqrt{1 + \cos \Delta\phi}}{M\omega^2 - k_\xi} \int_0^{2\pi/\omega} \sin(\omega t + \delta) \cdot \cos(\omega t + \Delta\phi) dt - \frac{rD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \lambda) \cdot \cos(\omega t + \Delta\phi) dt + \frac{\sqrt{1 - \cos \Delta\phi}}{M\omega^2 - k_\eta} \int_0^{2\pi/\omega} \sin(\omega t + \kappa) \cdot \sin(\omega t + \Delta\phi) dt - \frac{RD \sqrt{1 - \cos \Delta\phi}}{J\omega^2 - k_y l^2} \int_0^{2\pi/\omega} \sin(\omega t + \lambda) \cdot \sin(\omega t + \Delta\phi) dt - \frac{m^2 e^2 \omega^5 r^2}{2\pi k_o} \left[ \int_0^{2\pi/\omega} \sin \omega t \cdot \cos(\omega t + \Delta\phi) dt - \int_0^{2\pi/\omega} \sin(\omega t + \Delta\phi) \cdot \cos(\omega t + \Delta\phi) dt \right] \right] \quad (22)$$

Calculating integrals and substituting expressions (7) determining phase angle values  $\kappa$ ,  $\lambda$ ,  $\delta$  after rearrangements, the following is finely obtained:

$$\tilde{\Omega}_{w1} = \frac{-m^2 e^2 \omega^4}{2} \left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} - \frac{r^2}{k_o} \right] \sin \Delta\phi \quad (23)$$

In a similar fashion, it is possible to obtain for the second vibrator:

$$\tilde{\Omega}_{w2} = -\tilde{\Omega}_{w1} \quad (24)$$

An equation for the phase angle  $\Delta\varphi$  has the form:

$$\tilde{\Omega}_{w2} - \tilde{\Omega}_{w1} = 0 \quad (25a)$$

which is equivalent to:

$$\left[ \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} - \frac{1}{M\omega^2 - k_\xi} - \frac{r^2}{k_o} \right] \sin \Delta\phi = 0 \quad (25b)$$

Since the sign of the coefficient in square bracket in (25b) decides on the synchronisation character [2], therefore, as it is seen from this dependence form, too small rotational stiffness of the vibrators set placement  $k_o$  can lead to the reversal of sign of expression (25b) (which should be positive for over-resonance machines) and to the change of the stable value of the vibrators phase

angle  $\Delta\varphi$  from 0 (desired value) into  $\pm\pi$ . This would cause the vibrations decay in the working direction and undesirable movement in the transverse direction.

So, condition for proper synchronisation has in this case the form:

$$\frac{r^2}{k_o} + \frac{1}{M\omega^2 - k_\xi} < \frac{D^2}{J\omega^2 - k_y l^2} + \frac{1}{M\omega^2 - k_\eta} \quad (26)$$

## 5. Conclusions

The performed considerations enabled drawing some conclusions.

1<sup>o</sup> Individual mounting elasticity of each vibrator does not influence the way of their synchronisation.

2<sup>o</sup> Elasticity of the mounting the mutually stiff set of vibrators to the machine body in one linear or rotational direction can lead to a machine malfunction, which can be resolved by analysing dependencies (19) and (26).

## REFERENCES

- [1] I.I. B l e c h m a n, Vibratory Mechanics. Nauka, Moskwa 1994.
- [2] J. M i c h a l c z y k, Stany nieustalone nadrezonansowych maszyn wibracyjnych i ich wpływ na bezpieczeństwo pracy maszyn i urządzeń. Prace Komisji Nauk Techn. PAU **4** (2010).
- [3] J. M i c h a l c z y k, P. C z u b a k, Influence of Collisions with a Material Feed on Cophasal Mutual Synchronization of Driving Vibrators of Vibratory Machines. Journal of Theoretical and Applied Mechanics **48**, 1 (2010).
- [4] J. M i c h a l c z y k, Angular oscillations of vibratory machines of independent driving systems caused by a non-central direction of the exciting force operations. Archives of Mining Sciences 1 (2012).
- [5] T. B a n a s z e w s k i, Przyczyny nieprawidłowej samo-synchronizacji wibratorów w przesiewaczach. IX Sympozjum Techniki Wibracyjnej i Wibroakustyki, Kraków 1990.
- [6] Ch. H a y a s i, Nonlinear Oscillations in Physical Systems. McGraw-Hill, Inc. 1964.