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MODEL AND NUMERICAL ANALYSIS OF HARDENING PROCESS PHENOMENA FOR MEDIUM-CARBON STEEL

MODEL I ANALIZA NUMERYCZNA ZJAWISK PROCESU HARTOWANIA STALI ŚREDNIOWĘGLOWEJ

This paper refers to numerical modeling of thermal phenomena, phase transformations in solid state and mechanical phenomena occurring during hardening process of steel element (C45). In the algorithm heat transfer equation, equilibrium equations and macroscopic model of phase transformations basis of CCT diagrams are used. Coupling between basic phenomena of hardening process is considered, in particular the influence of latent heat on the temperature, and also thermal, structural and plastic strains – the transformation induced plasticity in the model of mechanical phenomena is taken into account as well.

Heat conductivity equation is used to estimate the temperature field in the process of heating and cooling. This equation is solved by finite element method in Galerkin formulation. Field of stresses and strains are obtained from solutions of finite element method equations of equilibrium in increment form. The influence of the temperature on material properties is also taken into account. To calculate of plastic strains the Huber-Mises condition with isotropic enhancement is used.

The method of calculating the phase transformation during heating applied by the authors uses data from the continuous heating diagram (CHT). Because start and finish of phase transformations strongly depends on the rate of heating or holding in specified temperature the dynamics curves A_{c1} and A_{c3} are used. The homogenization line of austenite determines the end of heating. The volume fraction of austenite during high rate of heating is determined by modified Koistinen-Marburger equation. The volume fractions of phase that emerge during cooling is determined by Avrami equation. The influence of austenisation temperature on the kinetics of transformations is taken into account. To calculate the increase of martensite content Koistinen-Marburger formula is used.

The numerical model was implemented in the Borland C++Builder 5.0 Environment. Using presented model, simulation of hardening for the cubic steel element is made. The element was heated by superficial source, and then cooled in water. The obtained results confirmed correctness of developed model and numerical algorithms.

Praca dotyczy modelowania numerycznego zjawisk cieplnych, przemian fazowych w stanie stałym oraz zjawisk mechanicznych towarzyszących procesom hartowania elementów ze stali średniowęglowej (C45). W algorytmie wykorzystano równanie przewodzenia ciepła, równania równowagi oraz makroskopowy model przemian fazowych oparty na wykresach CTPc. Zostały uwzględnione sprzężenia pomiędzy podstawowymi zjawiskami procesu hartowania, a w szczególności: wpływ ciepła przemiany fazowej na temperaturę, a w modelu zjawisk mechanicznych uwzględniono oprócz odkształceń termicznych, strukturalnych i plastycznych – również odkształcenia transformacyjne. Do wyznaczania pól temperatury w procesie nagrzewania i chłodzenia wykorzystano równanie różniczkowe przewodzenia ciepła. Równanie to rozwiązano metodą elementów skończonych w sformułowaniu Galerkina. Pola naprężeń i odkształceń uzyskano z rozwiązania metodą elementów skończonych równań równowagi w formie przyrostowej. Stałe termofizyczne uzależniono od temperatury. Odkształcenia plastyczne wyznacza się stosując warunek Hubera-Misesa ze wzmocnieniem izotropowym. W modelu przemian fazowych nagrzewania wykorzystuje się dane z wykresu CTPa. Ponieważ początek i koniec przemiany silnie zależy od prędkości nagrzewania bądź wytrzymania w określonej temperaturze, zastosowano dynamiczne krzywe A_{c1} i A_{c3}. Linia homogenizacji austenitu determinuje koniec nagrzewania. Do szacowania przyrostu fazy austenitycznej przy dużych szybkościach nagrzewania wykorzystano zmodyfikowany wzór Koistinena-Marburgera. Objętościowe udziały faz, mających miejsce podczas chłodzenia, szacuje się wzorem Avramiego. Przyrost udziału martenzytu wyznacza się natomiast zależnością Koistinena-Marburgera. Aplikacja będąca implementacją modelu numerycznego została wykonana w środowisku Borland C++Builder 5.0. Wykorzystując opracowany model wykonano symulację hartowania kostki stalowej. Element ten nagrzewano powierzchniowo, a następnie chłodzono wodą. Uzyskane wyniki potwierdzają poprawność zbudowanego modelu i algorytmów numerycznych.

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1. Introduction

Phase transformations of steel in solid state are basic element for technology of heat treatment. These processes are hard to experimental observation and mathematical description. There is a large number of parameters which have marked influence on the hardening process concerning both methods of heating and cooling. Sets of parameter have influences on hardening determinate progress in the numerical method, which are tools developments for modern process engineer. Rapid growth of processor capacity and creation modern research instruments, many types of numerical models of phase transformations were developed. These methods can be divided into the following types: a) macroscopic, based on Fe_3-C and CCT diagrams, and b) microscopic, analyzed growth of individual grains of phase on the basis of carbon diffusion and balance of energy [1, 2, 3, 4].

One of the most important problems in hardening process modeling is taking into account mutual interactions of thermal phenomena, phase transformations and mechanical phenomena. In mathematical description relations between proper models are very complicated, particularly when feedback is considered. In numerical model relationships of this type demand the multi-level recurrent algorithms or severe limitations of the time step (comp. Fig. 1).

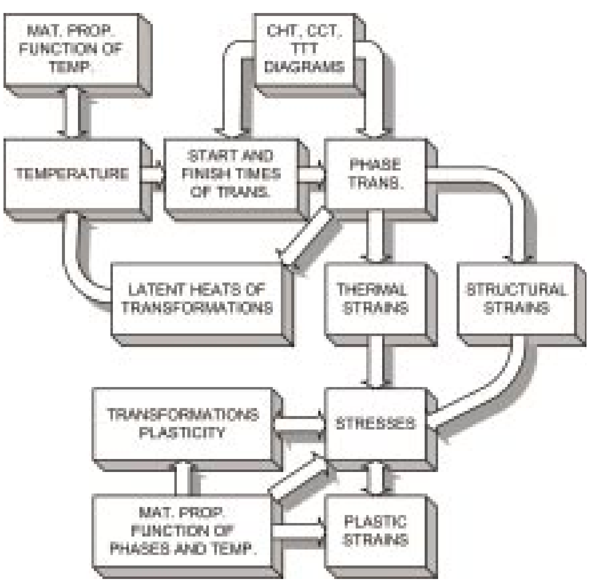

Fig. 1. The flow diagram of the model

Numerical analysis of heat treatment processes is very important problem. Many designer studios working for the industry (not only metallurgical) one facing this problem. However existing mathematical and numerical models do not take into account many important factors. For full analysis of heat treatment process the field of temperature, kinetics of phase transformations, tthermal and structural stresses should be incorporated.

2. Model of thermal phenomena

To determine of the temperature field, the partial differential equation description for unsteady heat transfer is used:

$$\nabla \cdot (\lambda \nabla T) - c\rho \frac{\partial T}{\partial t} + q_V = 0, \tag{2.1}$$

where: $\lambda = \lambda(T)$ is the thermal conductivity coefficient, $c = c(T)$ is the thermal capacity, ρ is the density and $q_V = q_V(x_\alpha, t)$ is the volumetric heat source coming from latent heat of transformations, x_α is the vector location of considered point.

During heat treatment of steels, phase transformations generate considerable amount of heat. Particularly, this phenomenon in transformation of austenite to pearlite is noticeable. In presented model latent heat of transformation as volumetric heat source in heat transfer equation is introduced.

$$\dot{q}_V = \sum_i H_i \frac{\Delta \eta_i}{\Delta t} \rho, \tag{2.2}$$

where: H_i [J/kg] is the heat of "i" transformation, $\Delta\eta_i$ is the volumetric increment of phase "i".

Changes of temperature, which are caused by latent heat of phase transformations, are most visible during cooling process of high carbon steel [5, 6]. In literature of specific heat of phase transformation the models, heat of transformation for particular phase (dependent of the temperature and chemical composition) are described [5]. In presented paper the results of analysis of Fe-C-Mn system and constant value of enthalpy for austenite-martensite transformation are used [5, 7]. This solution is approximated by the cubic polynomials:

$$\begin{aligned}
H_{\gamma\to\alpha}(T) &= 0.0006415 61T^3 - 0.593473214T^2 + 245.2464286T - 145423.0357 \text{ [J/kg]}\\
H_{\gamma\to P}(T) &= 0.00093038T^3 - 1.05366T^2 + 475.9625T - 212728.5714 \text{ [J/kg]}\\
H_{\gamma\to B}(T) &= 0.000775416T^3 - 0.816716071T^2 + 365.0160714T - 176705.35 \text{ [J/kg]}\\
H_{\gamma\to M}(T) &= 8.25 \cdot 10^4 \text{ [J/kg]},
\end{aligned} \tag{2.3}$$

where: $H_{\gamma\to\alpha}$ is the heat of austenite to ferrite transformation, $H_{\gamma\to P}$ is the heat of austenite to pearlite transformation, $H_{\gamma\to B}$ is the heat of austenite to bainite transformation, $H_{\gamma\to M}$ is the heat of austenite to martensite transformation, T is the temperature in °C.

Latent heat of transformation has influence especially on temperature changes and kinetics of phase transformations in the volumetric hardening, whereas it is insignificant in transformations during the surface hardening (Fig. 2).

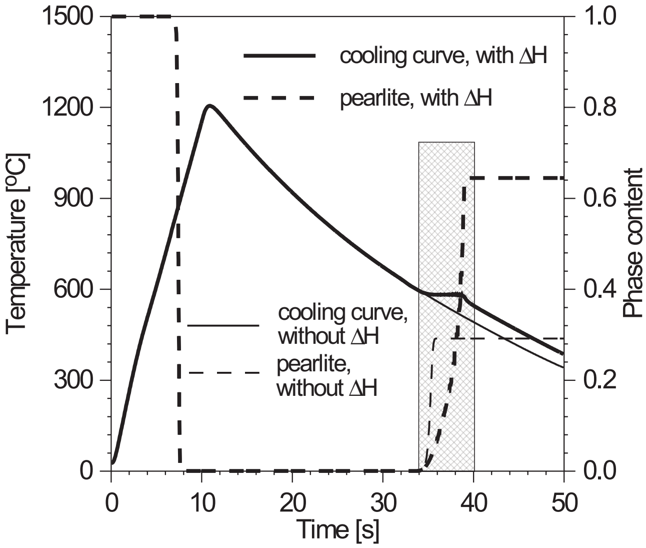

Fig. 2. Influence of latent heat of transformation on kinetic of phase transformation in the volumetric hardening, the middle node of cubical steel element

The equation completed boundary and initial conditions, and in finite element method of Galerkin formulation is solved [8, 9, 10].

3. Model of phase kinetics during heating

The proposed model of phase transformations during continuous heating uses data from CHT diagram [11,12]. This diagram presents decompositions of phases into austenite, which is formed during heating process. Because start and finish of phase transformation strongly depend on heating rate or time of holding, dynamical curves A_{c1} and A_{c3} are used. To define end of heating the curve of homogenization of austenite is used (Fig. 3). Above this curve can be observed non-advisable growth grains of austenite.

In every step the fraction of new phase is calculated on the basis of kinetics of the transformation modeled according to Johnson-Mehl-Avrami laws. Empirical Avrami equation, derived using an assumption of monophase of material, for heating process has the form [13]:

$$\tilde{\eta}_\gamma (T, t) = 1 - \exp\left(-b(T)\, t^{n(T)}\right), \tag{3.1}$$

where: $\tilde{\eta}_\gamma$ is a volume fraction of forming austenite, b and n are functions which depend on temperature as well as on start (t_s) and finish (t_f) times of transformation [9].

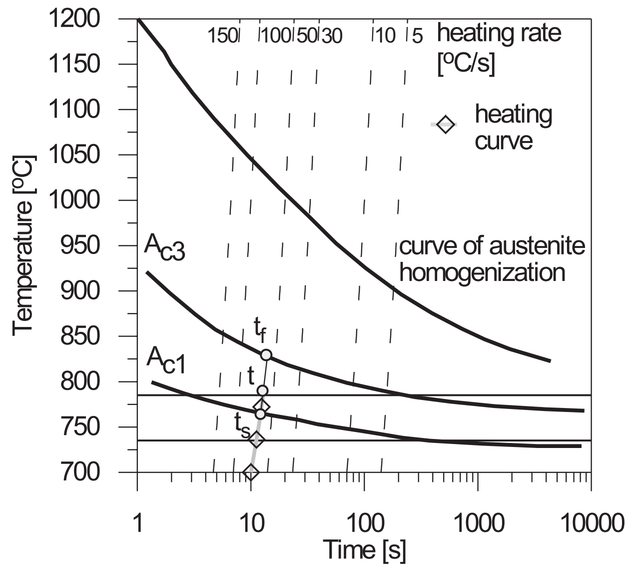

Fig. 3. The analysis of the diagram CHT, steel C45

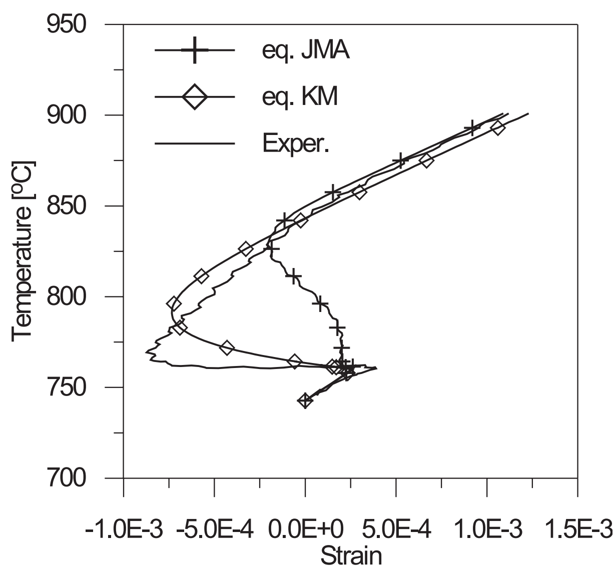

Fig. 4. The strains during austenite formation for heating rate 100°C/s

$$n(T) = 6.12733/\ln\left(\frac{t_f(T)}{t_s(T)}\right), b(T) = \frac{0.01005}{t_s^{n(T)}}. \tag{3.2}$$

To determinate the increment of austenite volume, particularly for high rate of heating, the modified Koistinen-Marburger equation is used (comp. Fig. 4) [3, 9]:

$$\tilde{\eta}_\gamma (T,t) = 1 - \exp\left(-k_\gamma \left(T_{s\gamma} - T\right)\right), k_\gamma = \frac{4{,}60517}{\mathrm{T}_{s\gamma} - T_{f\gamma}}, \tag{3.3}$$

where: $T_{s\gamma}$ is start temperature of austenite formation, $T_{f\gamma}$ is estimated finish temperature of transformation depended on rate of heating.

4. Model of phase kinetics during cooling

The volume fractions $\eta_{(.)}(T,t)$ growth during time of cooling is calculated by Avrami equations [9, 10, 13].

$$\eta_{(i)}(T,t) = \min\left\{\eta_{(i\%)}, \tilde{\eta}_\gamma - \sum_{j\neq i}\eta_j\right\}\cdot \cdot\left(1 - \exp\left(-b(T)\,t^{n(T)}\right)\right), \tag{4.1}$$

where: η_j is volume fraction of phase formed during cooling process, $\eta_{(i\%)}$ is final fraction of "i"-phase estimation of basis on CCT diagrams considered steel.

In presented model kinetics of phase transformation are determinate by Avrami equations. In paper the model, which determinate kinetics of individual phases with the use of spline functions is applied. If the phase transformation is first in cooling process and time of its start is known, then finish time of last phase (t_f) is calculated by function based on prediction the volume fraction and finish time of first transformation (analysis of CCT diagrams):

$$t_f(t,\eta_\%,t_s) = \exp\left(\frac{-A\cdot(\ln(t_s) - \ln(t))}{B(\eta_\%)}\right)\cdot t_s. \tag{4.2}$$

If the phase transformation is last and time of its finish is known, then start time of first transformation (t_s) is determinated by equation:

$$t_s\left(t,\eta_\%,t_f\right) = \left(\frac{A}{t^{B(\eta_\%)}\cdot t_f}\right)^{\left(\frac{B(\eta_\%)}{A-B(\eta_\%)}\right)}, \tag{4.3}$$

where: $\eta_\%$ is estimated volume fraction of the last phase.

Whereas, if the phase transformation takes place between another, then start and finish time of transformation basis on time of end and volume fraction previous transformation, and predictions time of end and volume fractions considered phase is determined:

$$t_s(t_1,\eta_{(1\%)},t_2,\eta_{(2\%)}) = = \sqrt[N\left(\eta_{(1\%)},\eta_{(2\%)}\right)]{\left(\frac{A}{t_1^{B\left(\eta_{(1\%)}\right)}\cdot\exp(-A\cdot\ln(t_2))^{B\left(\eta_{(1\%)}\right)}}\right)}. \tag{4.4}$$

The coefficients A(), B() and N() are calculated using following formulas:

$$N(\eta_{(1\%)},\eta_{(2\%)}) = \left(\frac{-A}{\mathrm{B}(\eta_{(2\%)})} + 1\right)\left(\frac{\mathrm{B}(\eta_{(1\%)})}{A - \mathrm{B}(\eta_{(1\%)})}\right) + 1 \tag{4.5}$$

$$A = \exp\left(\ln\left(\frac{\ln\left(1-\eta_f\right)}{\ln(1-\eta_s)}\right)\right),\ B(\eta_\%) = \ln\left(\frac{\ln(1-\eta_\%)}{\ln(1-\eta_s)}\right). \tag{4.6}$$

Among considerable factors deciding in primary measure about course of transformation are stress conditions. In this model modification of equations describing kinetics of phase transformations are introduced [11,14,15]. Influence of the mechanical phenomena on changes of material microstructure is taken into account. This phenomenon is especially noticeable during martensite transformation. This effect describes as dependent the martensite start temperature on normal and tangential stresses (comp. Fig. 5).

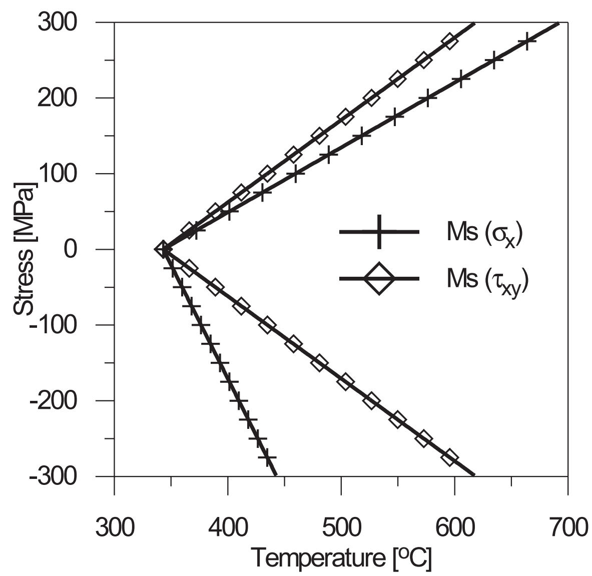

Fig. 5. Influence of stresses (normal σ_x and tangential τ_{xy}) on martensite start temperature (M_s)

The phase transformation during the high-rate cooling (transformation of austenite to martensite) is determined by classic form of Koistinen-Marburger equation (for $T < M_s$) [3, 11, 14, 15]:

$$\eta_M(T,t) = \left(\tilde{\eta}_\gamma - \sum_{i\neq M}\eta_i\right) \left(1 - \exp\left(-k\left(M_S - T + A_M\sigma_{eff} + B_M\sigma_a\right)\right)\right), k = 0.01537, \tag{4.7}$$

where: $\sigma_a = (\sigma_{ii})/3$ – average stress, $\sigma_{eff} = \sqrt{3/2\left(\boldsymbol{\sigma}^D \cdot \boldsymbol{\sigma}^D\right)}$ – effective stress, $\boldsymbol{\sigma}^D$ – deviatoric stresses, A_M and B_M coefficients for C45 steel are equal to $A_M = 1.25 \times 10^{-6}$ [K/Pa], $B_M = 0.75 \times 10^{-6}$ [K/Pa] [11].

According to eq. 4.7 and Fig. 5 it can be noticed, that temperature M_s lowers when average stress is less than zero, otherwise M_s raises. Effective stress always increases temperature M_s, so the austenite – martensite transformation is accelerated.

The increase of isotropic strain resulting from the temperature and phase transformation ($d\varepsilon^{Tph} = d\varepsilon^T + d\varepsilon^{ph}$) during heating and cooling is described by formula [9]:

$$d\varepsilon^T = \sum_i \alpha_i(T)\,\eta_i dT, d\varepsilon^{ph} = \sum_i \varepsilon_i^{ph}(T)\,d\eta_i \qquad (4.8)$$

where: α_i is a thermal expansion coefficients for "i" phase, $\varepsilon_i^{ph} = \delta V_i/(3V)$ is a structural expansion coefficients for "i" transformation. Experimental research and numerical simulations – dilatometric curves, assign these values (thermal and structural expansion coefficients). For the C45 steel they have the values: 2.178, 1.534, 1.534, 1.171 and 1.36 ($\times 10^{-5}$) [1/K] and −1.986, 1.534, 1.534, 4.0 and 6.5 ($\times 10^{-3}$) for austenite, ferrite, pearlite, bainite and martensite respectively [9].

The kinetics of phase transformations during cooling strongly depend on austenisation temperature, therefore model, which exploits two CCT diagrams for C45 steel (880°C and 1050°C austenisation temperature, Fig. 6), was developed [16, 17].
Displaced diagrams with respect to each other are used to determine intermediate diagrams using linear approximation [9].

$$\begin{aligned}
t_j(T_{Aust}) &= \gamma_{Aust} t_j^{1050} + (1 - \gamma_{Aust})\, t_j^{880} \\
T_j(T_{Aust}) &= \gamma_{Aust} T_j^{1050} + (1 - \gamma_{Aust})\, T_j^{880} \qquad (4.9)\\
\eta_{(i\%)j}(T_{Aust}) &= \gamma_{Aust} \eta_{(i\%)j}^{1050} + (1 - \gamma_{Aust})\, \eta_{(i\%)j}^{880}.
\end{aligned}$$

Parameter γ_{Aust} is described by formula [9]:

$$\begin{aligned}
\gamma_{Aust} &= 0 && \text{for } T_{\max} < 880°C\\
\gamma_{Aust} &= \tfrac{T_{\max} - 880}{1050 - 880} && \text{for } 880°C < T_{\max} < 1050°C\\
\gamma_{Aust} &= 1 && \text{for } T_{\max} > 1050°C.
\end{aligned} \qquad (4.10)$$

The presented numerical model for numerical simulations of strains during phase transformation of heating and cooling for different austenisation temperatures (Fig. 7) is used.

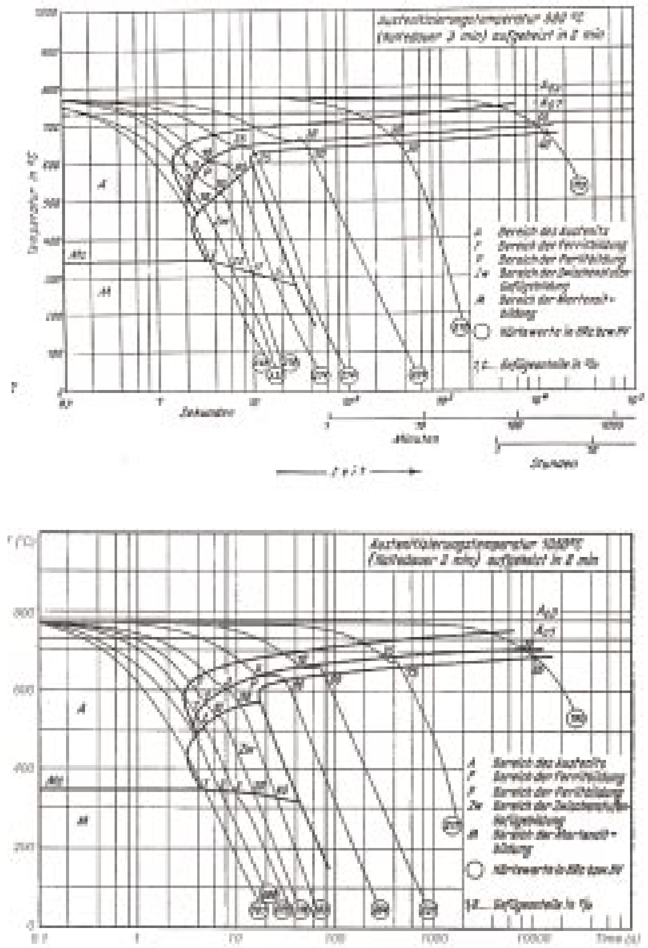

Fig. 6. The CCT diagrams of the C45 steel for different austenitisation temperatures: a) 880°C [16], b) 1050°C [17]

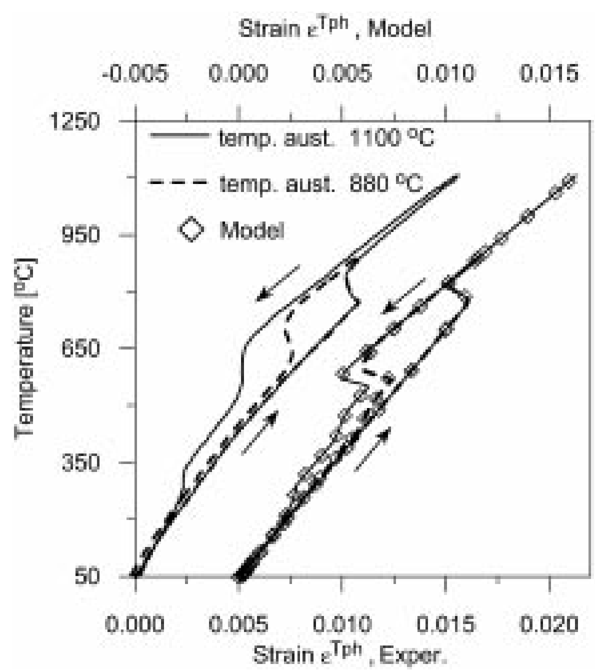

Fig. 7. Influence of austenitisation temperature on the dilatometric curves. (Experimental researches and numerical simulations)

5. Model of mechanical phenomena

The heat treatment process of steel elements causes a sudden change in stress fields. This change is the result of different types of strains. In the presented model, except elastic ($\boldsymbol{\varepsilon}^{\mathbf{e}}$), thermal ($\boldsymbol{\varepsilon}^{\mathbf{T}}$), structural ($\boldsymbol{\varepsilon}^{\mathbf{ph}}$) and plastic ($\boldsymbol{\varepsilon}^{\mathbf{pl}}$) strains, the transformations plasticity ($\boldsymbol{\varepsilon}^{\mathbf{tp}}$) are taken into accounts as well ($\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathbf{e}} + \boldsymbol{\varepsilon}^{\mathbf{T}} + \boldsymbol{\varepsilon}^{\mathbf{ph}} + \boldsymbol{\varepsilon}^{\mathbf{pl}} + \boldsymbol{\varepsilon}^{\mathbf{tp}}$) [8, 9, 11].

The constitutive relations are assumed in the form:

$$\Delta\boldsymbol{\sigma} = \mathbf{D} \circ \Delta\boldsymbol{\varepsilon}^{\mathbf{e}} + \Delta\mathbf{D} \circ \boldsymbol{\varepsilon}^{\mathbf{e}}, \tag{5.1}$$

where: $\boldsymbol{\sigma}$ is tensor of stress, $\mathbf{D}$ is tensor of material constants.

In presented model increment of plastic strain is determined by nonisothermic plastic flow law:

$$\dot{\boldsymbol{\varepsilon}}^{\mathbf{pl}} = \dot{\lambda}\frac{3\boldsymbol{\sigma}^{\mathbf{D}}}{2\bar{\sigma}} \tag{5.2}$$

in which scalar plastic multiplier is calculated by (isotropic hardening):

$$\dot{\lambda} = 2\bar{\sigma}\frac{3\boldsymbol{\sigma}^{\mathbf{D}}\left(\mathbf{D}\left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{T} - \dot{\boldsymbol{\varepsilon}}^{ph} - \dot{\boldsymbol{\varepsilon}}^{tp}\right) + \dot{\mathbf{D}}\boldsymbol{\varepsilon}^{e}\right) - 2\bar{\sigma}H_{T}^{\bar{\sigma}}\dot{T}}{9\boldsymbol{\sigma}^{\mathbf{D}}\left(\mathbf{D}\boldsymbol{\sigma}^{\mathbf{D}}\right) + 4\kappa\bar{\sigma}\sigma_{ef}^{D}}, \tag{5.3}$$

where: κ is the hardening modulus, $H_{T}^{\bar{\sigma}}$ is the thermal softening modulus described by:

$$H_{T}^{\bar{\sigma}} = \frac{\Delta\bar{\sigma}}{\Delta T} + \frac{\Delta\kappa}{\Delta T}\varepsilon_{eff}^{pl}. \tag{5.4}$$

The actual level of effective stress ($\bar{\sigma}$) is depended on temperature and composition of phase:

$$\bar{\sigma}\left(T, \sum_i \eta_i, \varepsilon_{eff}^{pl}\right) = \bar{\sigma}_0\left(T, \sum_i \eta_i\right) + \kappa(T)\,\varepsilon_{eff}^{pl}, \tag{5.5}$$

where: $\bar{\sigma}_0$ is the yield point, ε_{eff}^{pl} is the effective plastic strain.

The yield point depends on volume fractions of phase, is completed by nonlinear influence of hard phase (martensite) [11]:

$$\bar{\sigma}_0\left(T, \sum_i \eta_i\right) = \sum_{i\neq M}\eta_i\bar{\sigma}_i(T) + f(\eta_M)\,\bar{\sigma}_M(T), \tag{5.6}$$

where: $\bar{\sigma}_i$, $\bar{\sigma}_M$ are the yield points of particular phases [18], values of function $f = f(\eta_M)$ is calculated following:

$$\begin{aligned} &f(\eta_M) = \eta_M\left(\zeta + 2(1-\zeta)\,\eta_M - (1-\zeta)\,\eta_M^2\right), \\ &\zeta = 1.383\cdot\bar{\sigma}_\gamma(T)/\bar{\sigma}_M(T). \end{aligned} \tag{5.7}$$

Transformations induced plasticity ε^{tp} are non-reversible changes observed during phase transformations occur under loading (in state of stress). In the models based on Greenwood-Johnson mechanism transformations induced plasticity is described by [9, 16, 19]:

$$\dot{\boldsymbol{\varepsilon}}^{tp} = \sum_i \frac{3}{2}K_i F(\eta_i)\,\frac{\boldsymbol{\sigma}^D}{\bar{\sigma}}\dot{\eta}_i \tag{5.8}$$

where: $K_i = \frac{5}{6}\frac{\delta V_i}{V}$ is a parameter depended on change of volume during growth of "i" phase, $F(\eta_i) = -\ln(\eta_i)$ (Leblond [11, 19]) is a function which determine kinetics of transformation plasticity.

6. Experimental research

The experimental research concerning the phase transformation during continuous heating and cooling for carbon steel was made. The objects of research were specimens made of rolled round iron from C45 steel (norm PN-93/H-84019). The experimental research was done on two thermal cycle simulators: SMITWELD TCS 1405 (perpendicular specimens measuring 11×11×55 mm, with center hole ϕ4, for high cooling rate), which is the property of Institute of Mechanics and Machine Design of Technical University of Częstochowa and dilatometer DIL805 (cylindrical specimens ϕ4/2×10 mm and for the rate of cooling ⩽50°C/s ϕ4/3·10 mm), which is on equipment of Institute for Ferrous Metallurgy of Gliwice. In the experimental research, phase transformations during the thermal cycles of steel 45 exposed to quick heating and cooling with different rates (heating - 100°C/s, cooling – 10, 20, 30, 50, 100, 200 and 250°C/s), were investigated.

The research dealt with:

- phase transformations of the austenite steel C45 during the simulated hardening cycle (dilatometric tests),
- determination of thermal and structural expansion coefficients of the individual phases of the steel,
- the influence of the austenisation temperature on kinetics of transformations.

The results obtained from experimental analysis were applied in numerical model, and some were used as a comparative material to verify the model (comp. Fig. 8).

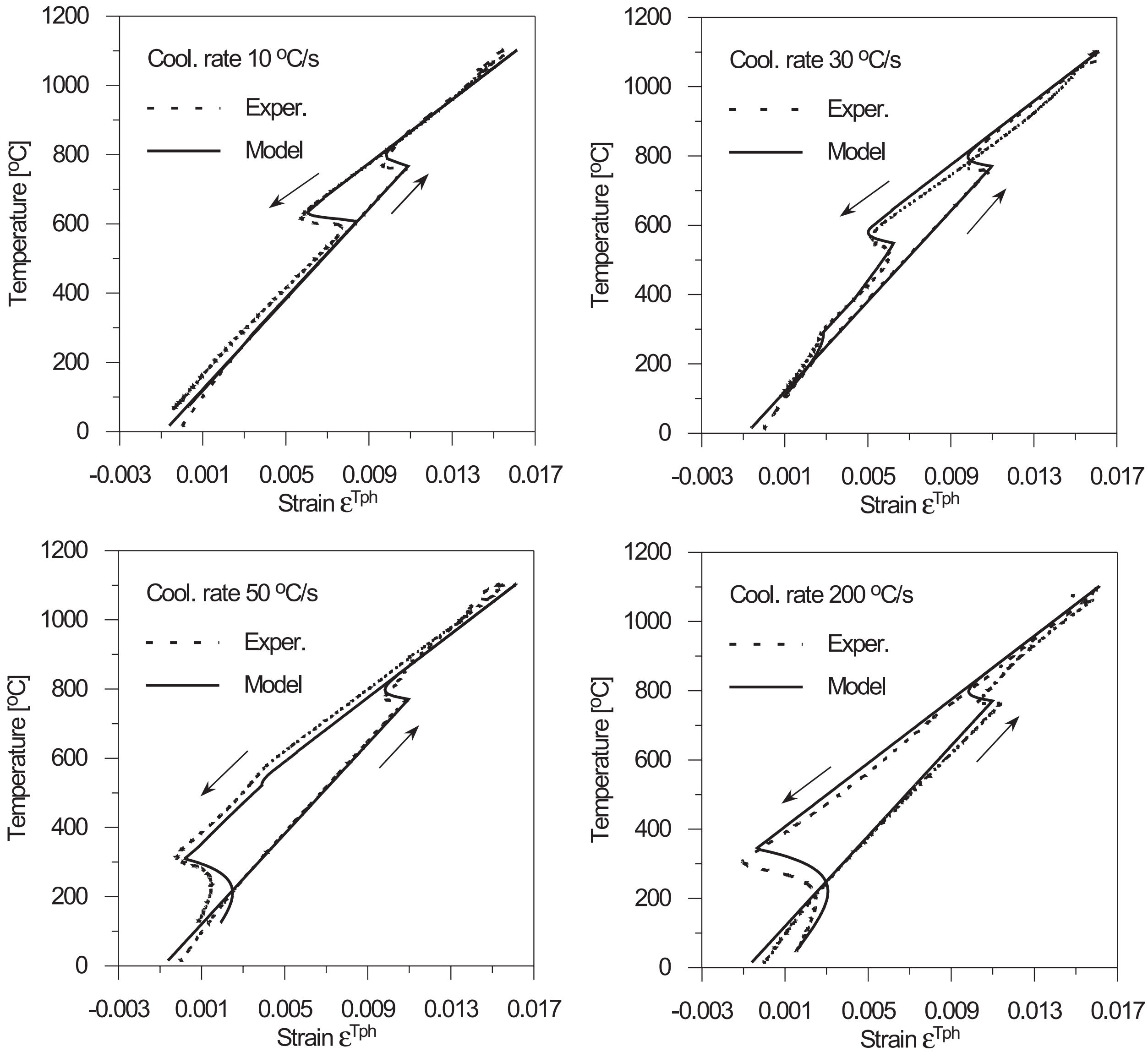

Fig. 8. Experimental and simulated dilatometric curves: a) 10°C/s, b) 30°C/s, c) 50°C/s, d) 200°C/s

7. Numerical simulation

The numerical simulation of the hardening process was made for the cubic steel element (dimensions 0.035×0.035×0.035 [m]). Heating was modeled by superficial heat source (Neumann condition), taking $q_n = 10^6$ [W/m^2]. After obtained maximum temperature equal 1673 K in corners (temperature in the middle of cube in this time was 961 K, A_{c1} = 1058 K), the element was cooled in water of temperature 333 K (60°C). Simulation of cooling was finished after leveling cube and water temperatures. The temperature dependence of convective heat transfer coefficient was described as a piecewise linear function. Values of these coefficients were determined in the experimental research [20].

In presented simulation of stresses was assumed, that values of Young's and tangential modulus were *constant*, value of tangential modulus was equal to: $E_t = 0.1 \times E$. Static yield point was determined from values of experimental research from paper [18]. These values were approximated by piecewise linear function.

The analysis of numerical simulation indicated that hardening element, which was made from C45 steel, was not full hardened (see Figs. 9 and 10). Because the water temperature was increased and the hardenability of steel was modest, the through hardening has not occurred.

For this hardening the maximum level of effective stresses were 500 [MPa] and the value of maximal effective plastic strains was equal 0.008 (comp. Figs. 11–13).

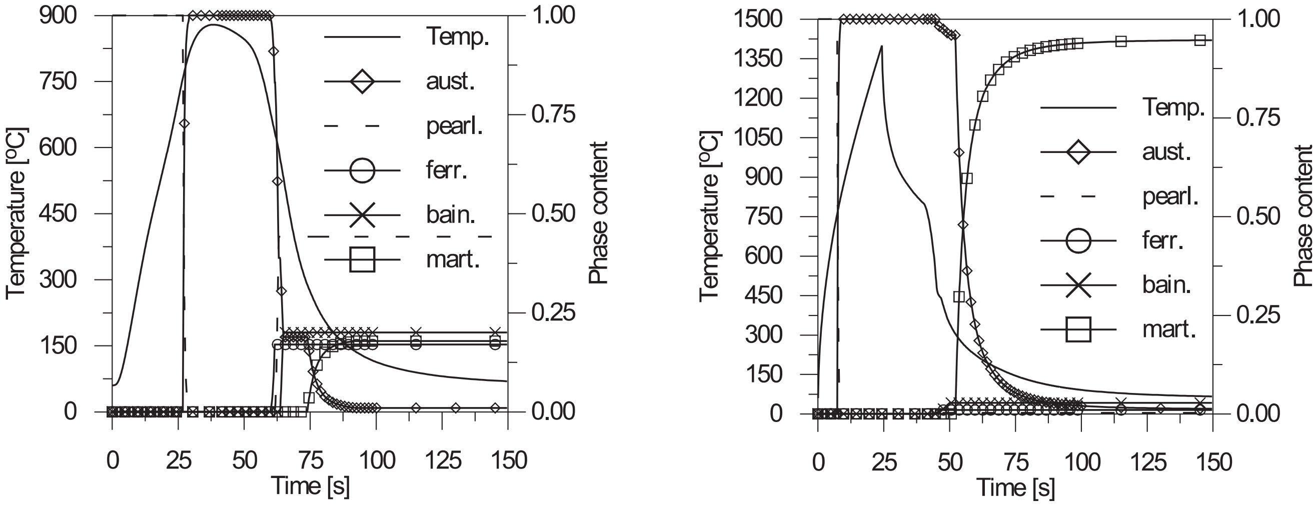

Fig. 9. The kinetics of phase transformations and temperature: a) in the middle node, b) in the corner node

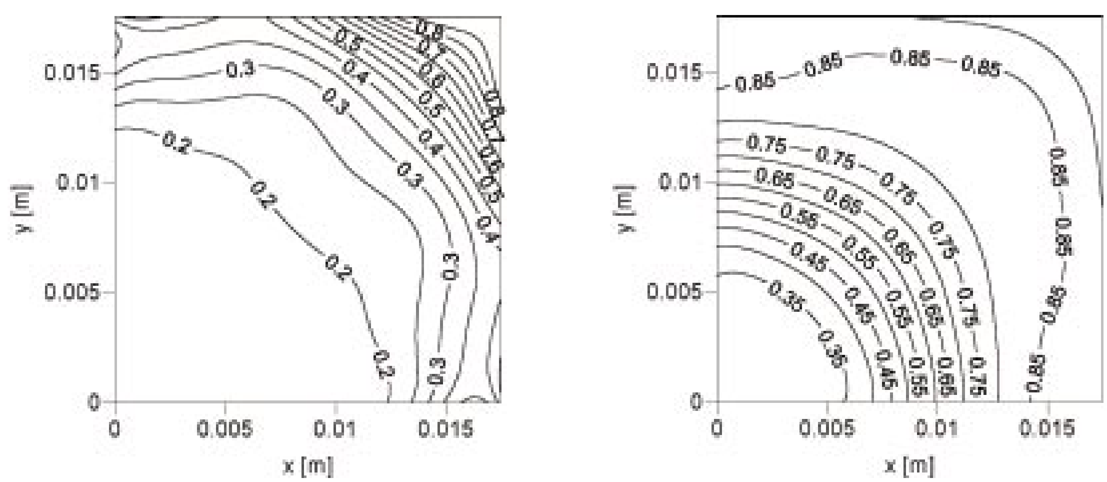

Fig. 10. The field of martensite: a) cross-sections, b) external surface

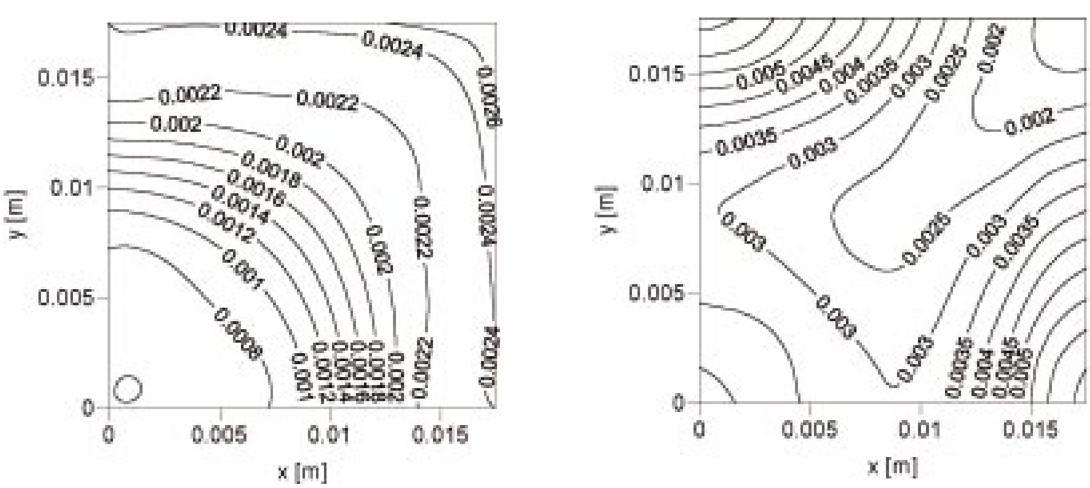

Fig. 11. The strains on external surface: a) the structural strains, b) the effective transformations plasticity

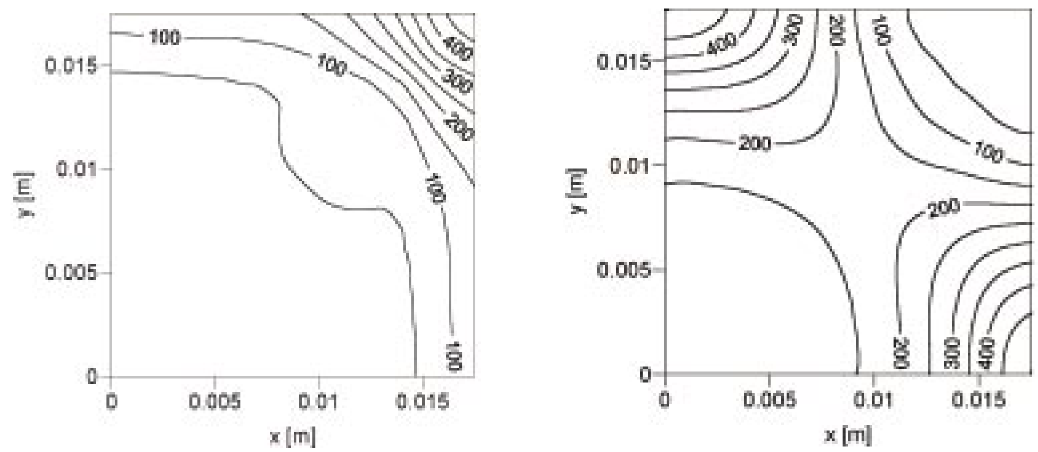

Fig. 12. The effective stresses: a) cross-sections, b) external surface

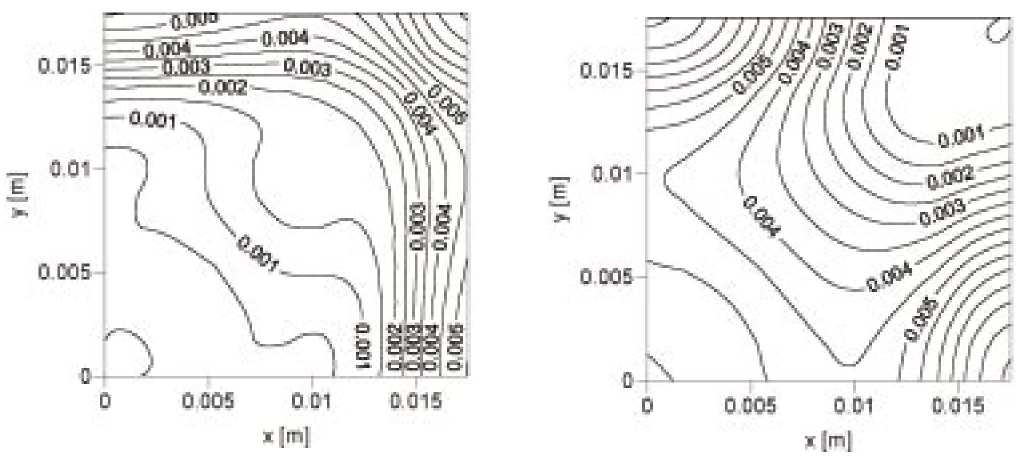

Fig. 13. The effective plastic strains: a) cross-sections, b) external surface

8. Conclusions

On the basis of the presented numerical and experimental research, the following conclusions are drawn:

- The results of the numerical model of phase transformations are in good conformity with the results of the experimental researches (Fig. 8).
- The thermal and structural expansion coefficients of the individual phases of the C45 steel determined from the dilatometric tests allow to calculate the strains of heat-treated steel elements.
- Developed 3D model allows to simulate the hardening processes for free forms of machine elements.
- Taking into account latent heats of transformation change in great agreement the kinetics of phase during hardening (Fig. 2).
- The calculation of volumetric increase of austenite for high rate of heating from Avrami equation is not accurate, therefore the modified Kointinen-Marburger equation is used (Fig. 4).
- The kinetics of phase transformations during cooling strongly depends on austenisation temperature. This phenomenon in the presented model is taken into account.
- If more diagrams CCT for different heating parameters are used, then presented model will be more accurate.
- The results of numerical simulation should be verified experimentally.

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